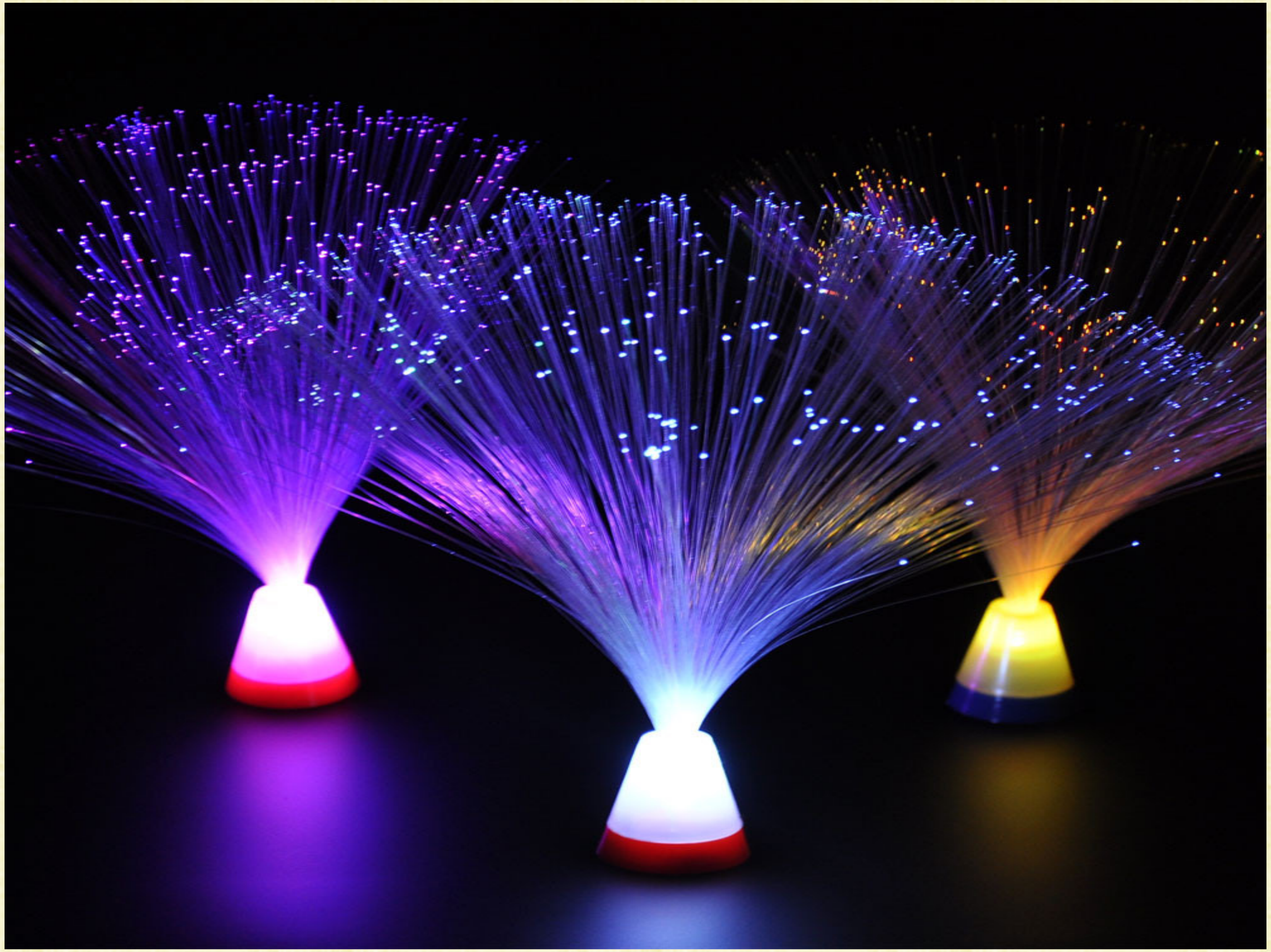
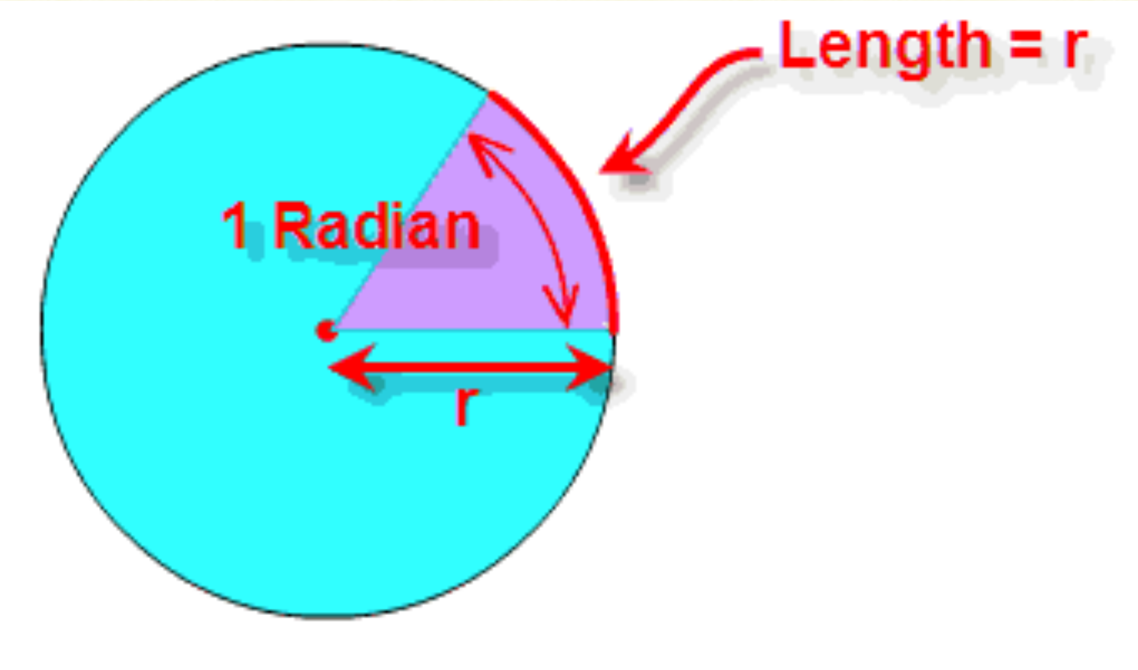
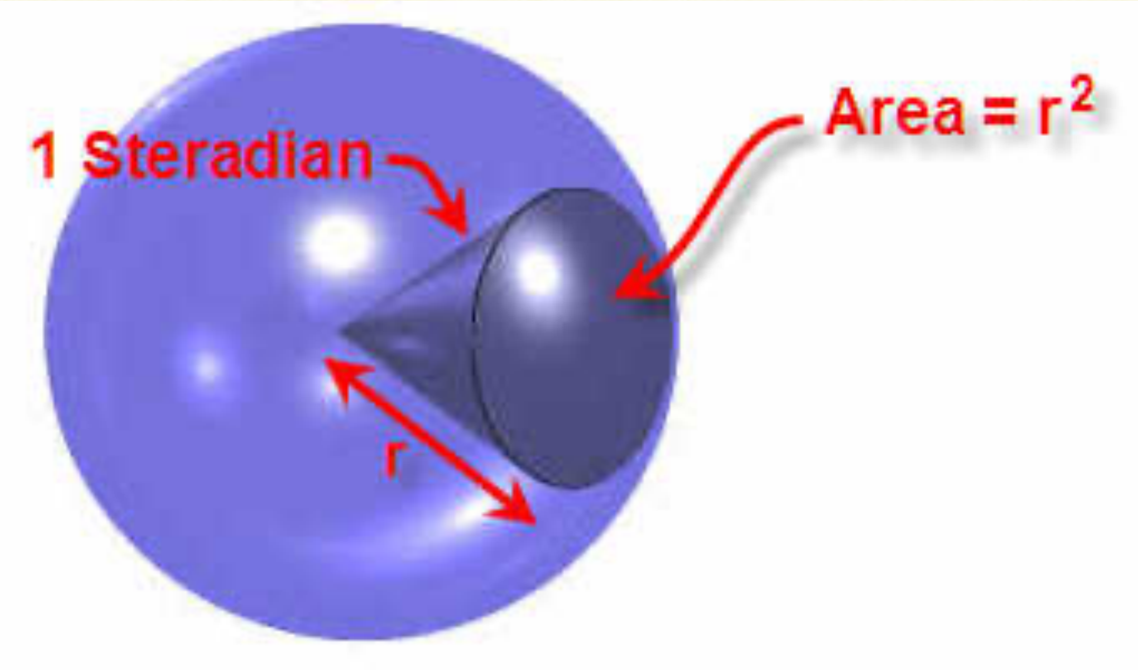
Optics



Solid Angle

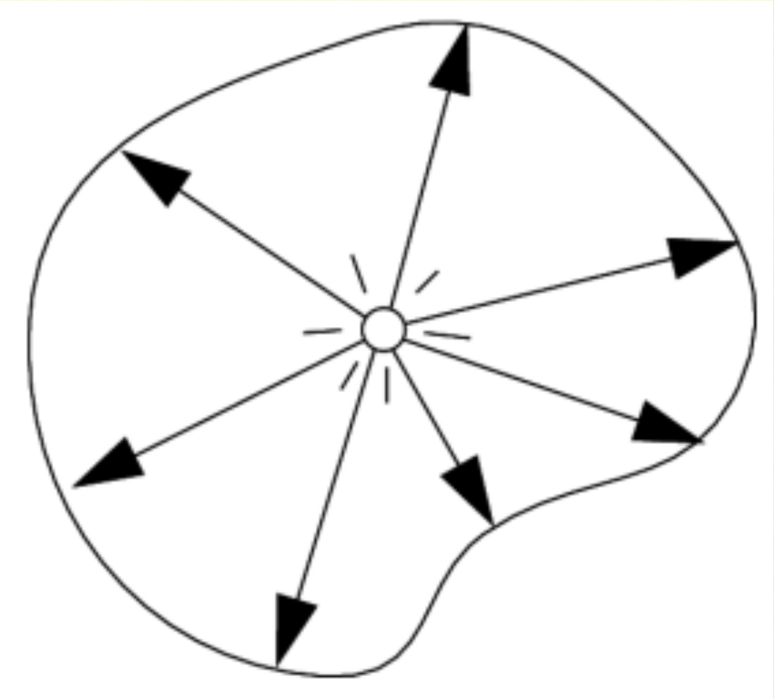
- A 2D angle in 3D, defined by a point and a surface patch (measured in steradians)
- Angles have $\theta = \frac{l_{arc}}{r}$, and solid angles have $\omega = \frac{A_{on\ the\ sphere\ surface}}{r^2}$
- Circumference of a circle is $C = 2\pi r$, so a circle has 2π radians
- Surface area of a sphere is $4\pi r^2$, so a sphere has 4π steradians



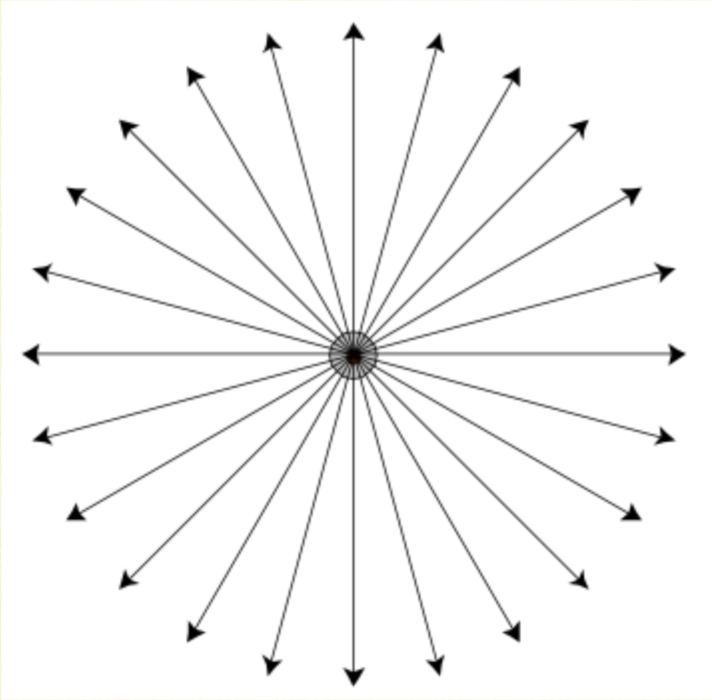


Radiant Intensity from a Light Source

- Power per unit solid angle $I(\omega) = \frac{d\Phi}{d\omega}$
  - Φ is the light source power (in watts = joules per second)
- Anisotropic light source: I varies across the light (as a function of ω)
- Isotropic point light: integrate $d\Phi = I d\omega$ to obtain $\Phi = \int_{sphere} I d\omega = 4\pi I$



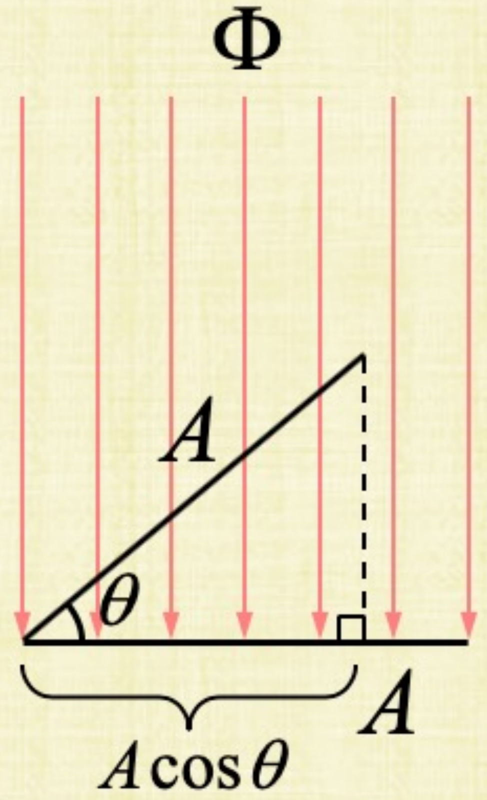
anisotropic light source

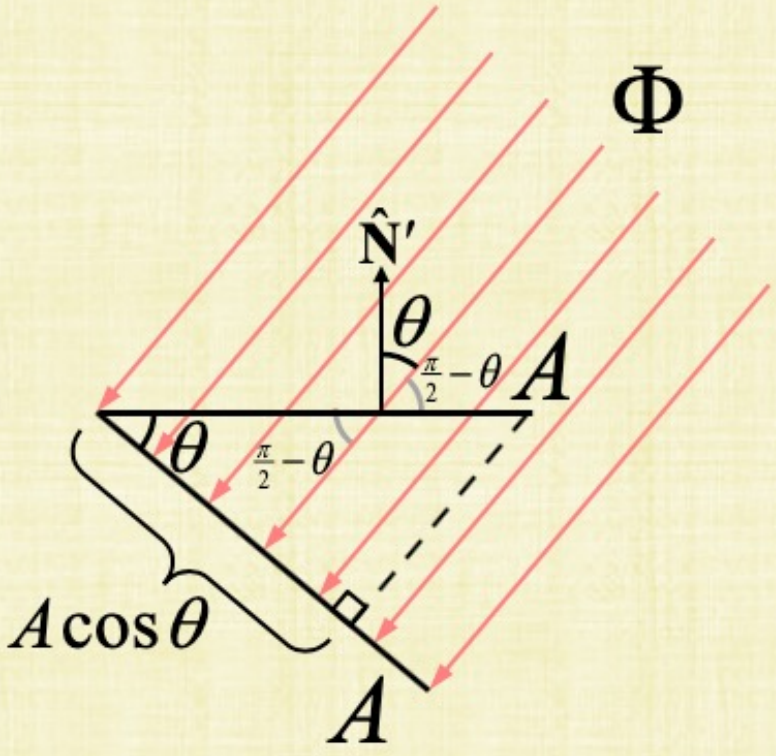


isotropic point light

Irradiance onto a Surface

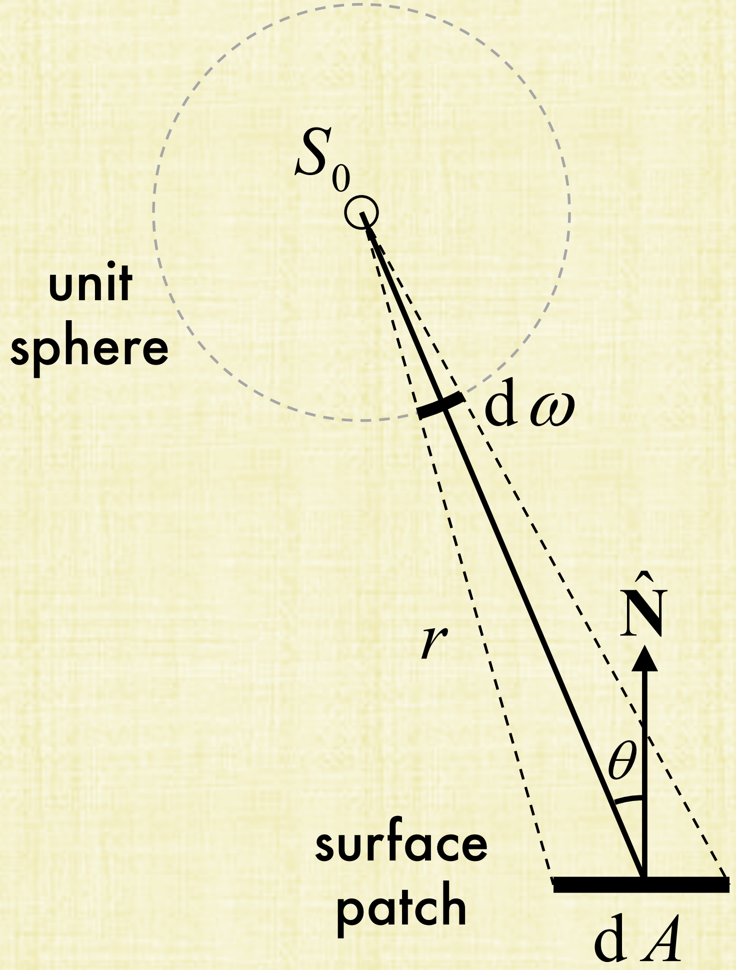
- Power per unit surface area $E = \frac{d\Phi}{dA}$
- Given $E_{flat} = \frac{\Phi_{flat}}{A}$, note that $E_{tilted} = \frac{\left(\frac{A cos\theta}{A}\right)\Phi_{flat}}{A} = E_{flat} cos\theta$
- Irradiance decreases as you tilt the surface, since less photons hit per unit surface area





Solid Angle vs. Cross-Sectional Area

- The (orthogonal) cross-sectional area is $dA\ cos\theta$ (from the previous slide)
- So, $d\omega = \frac{dA_{sphere}}{r^2} = \frac{dA\ cos\theta}{r^2}$ (solid angle varies with tilting θ and distance r)



Area Lights

- Light power is emitted per unit area (not from a single point)
- The emitted light goes in various directions (measured with solid angles)

- Break an area light up into (infinitesimally) small area chunks
- Each area chunk emits light into each of the solid angle directions
  - i.e. radiant intensity per area chunk
- Each emitted direction also has a cosine term (similar to irradiance)

- Radiance – radiant intensity per area chunk

$$L = \frac{dI}{dA\cos\theta_{light}} \left(= \frac{d^2\Phi}{d\omega\, dA\cos\theta_{light}} = \frac{dE}{d\omega\cos\theta_{light}} \right)$$

Objects act as Area Lights

- Light comes from all visible objects in the world (not just from light sources)
- Each area chunk of each object acts as a source of light (i.e. as a light source)

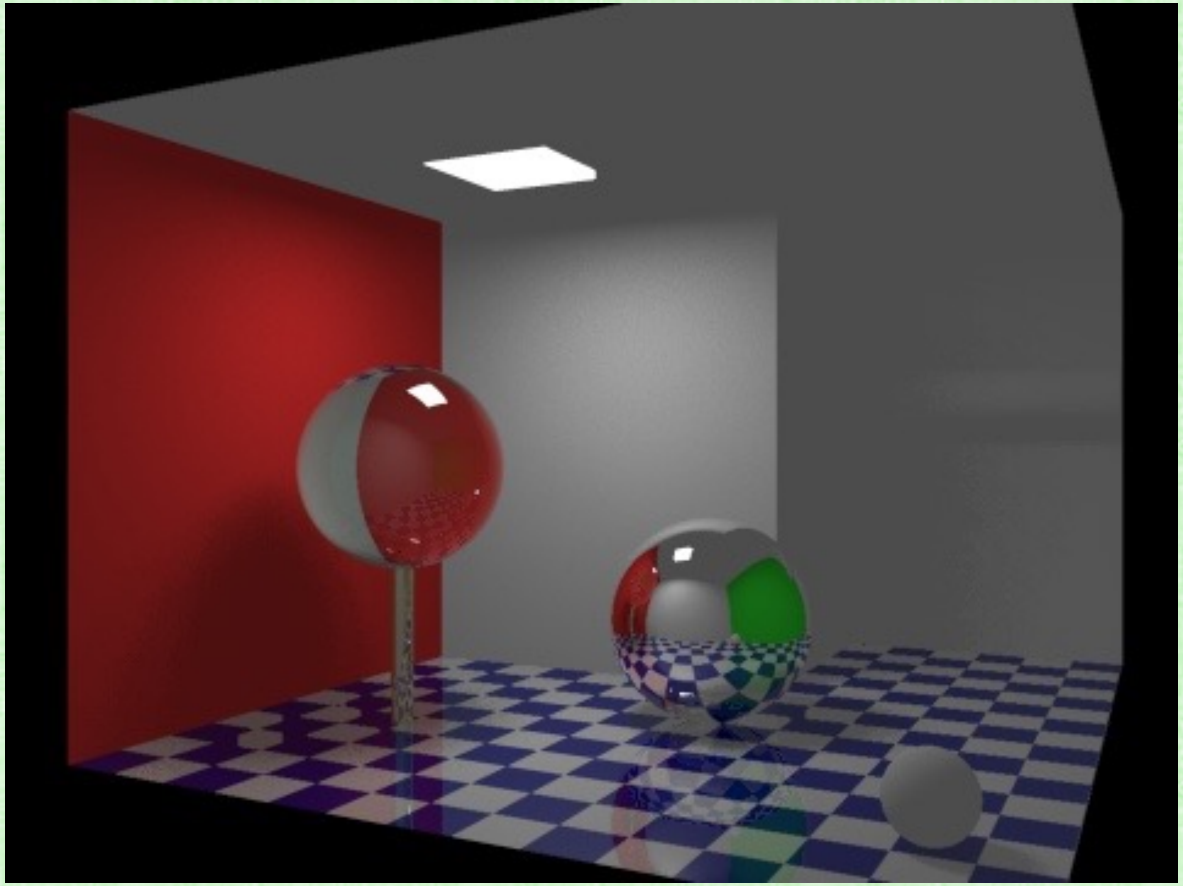
- A tree shines light onto the car; then, the car shines light onto the camera:

Objects act as Area Lights

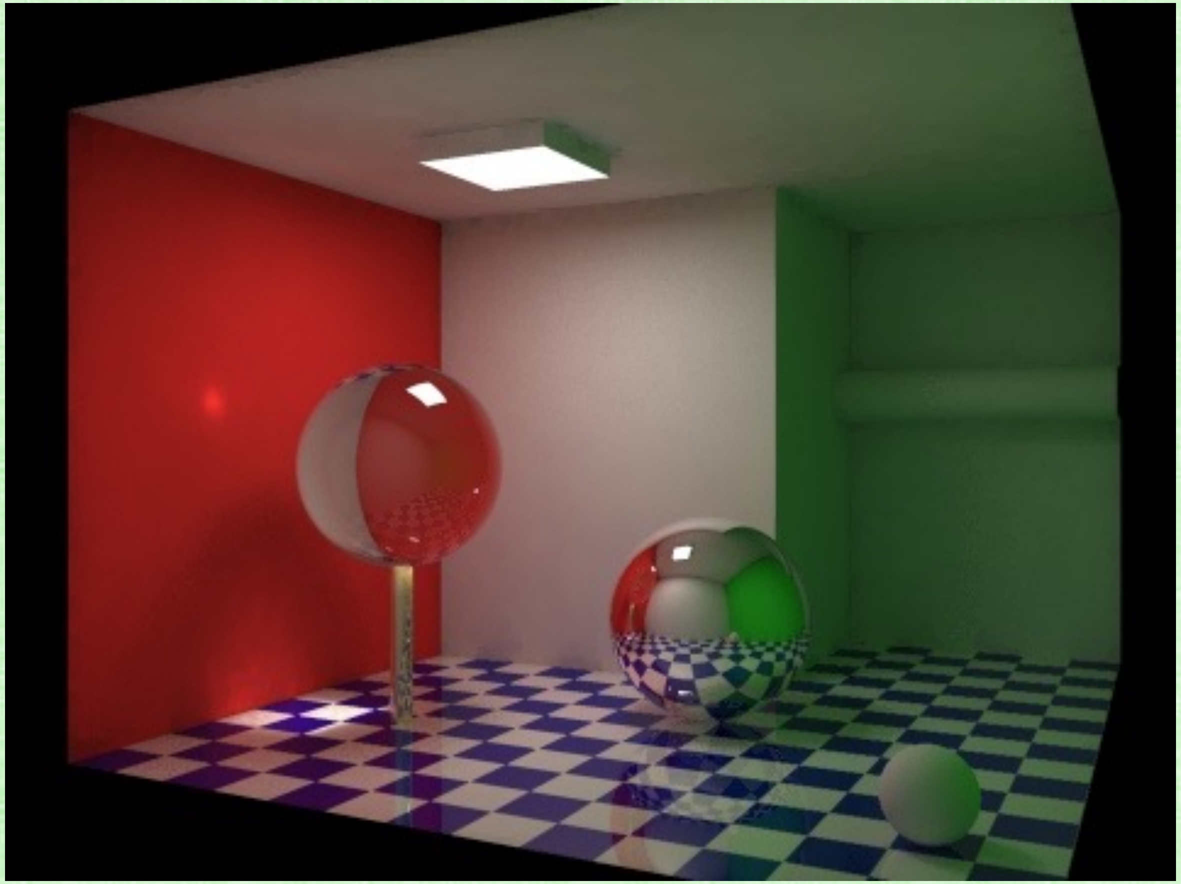
- The red paper shines red light onto the statue (color bleeding); then, the statue shines red light onto the camera:

Objects act as Area Lights

- It’s not good enough to only look for light along shadow rays (even though, it’s pretty good)



using light only from shadow rays



using light from shadow rays and objects

Measuring Incoming Light

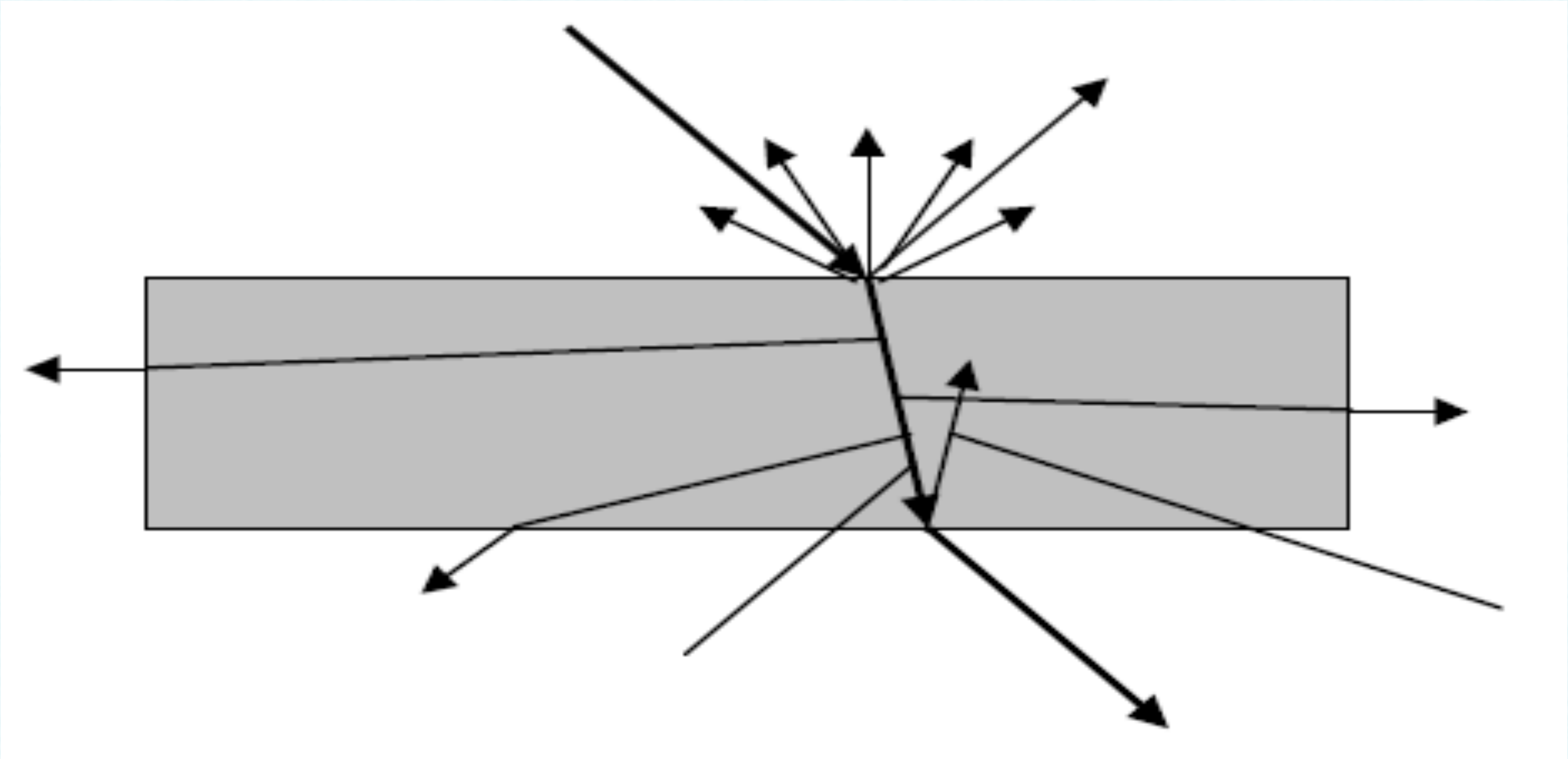
- Light Probe: a small reflective chrome sphere
- Photograph it, in order to record the incoming light (at its location) from all directions

Using the (measured) Incoming Light

- The (measured) incoming light can be used to render a synthetic object (with realistic lighting)

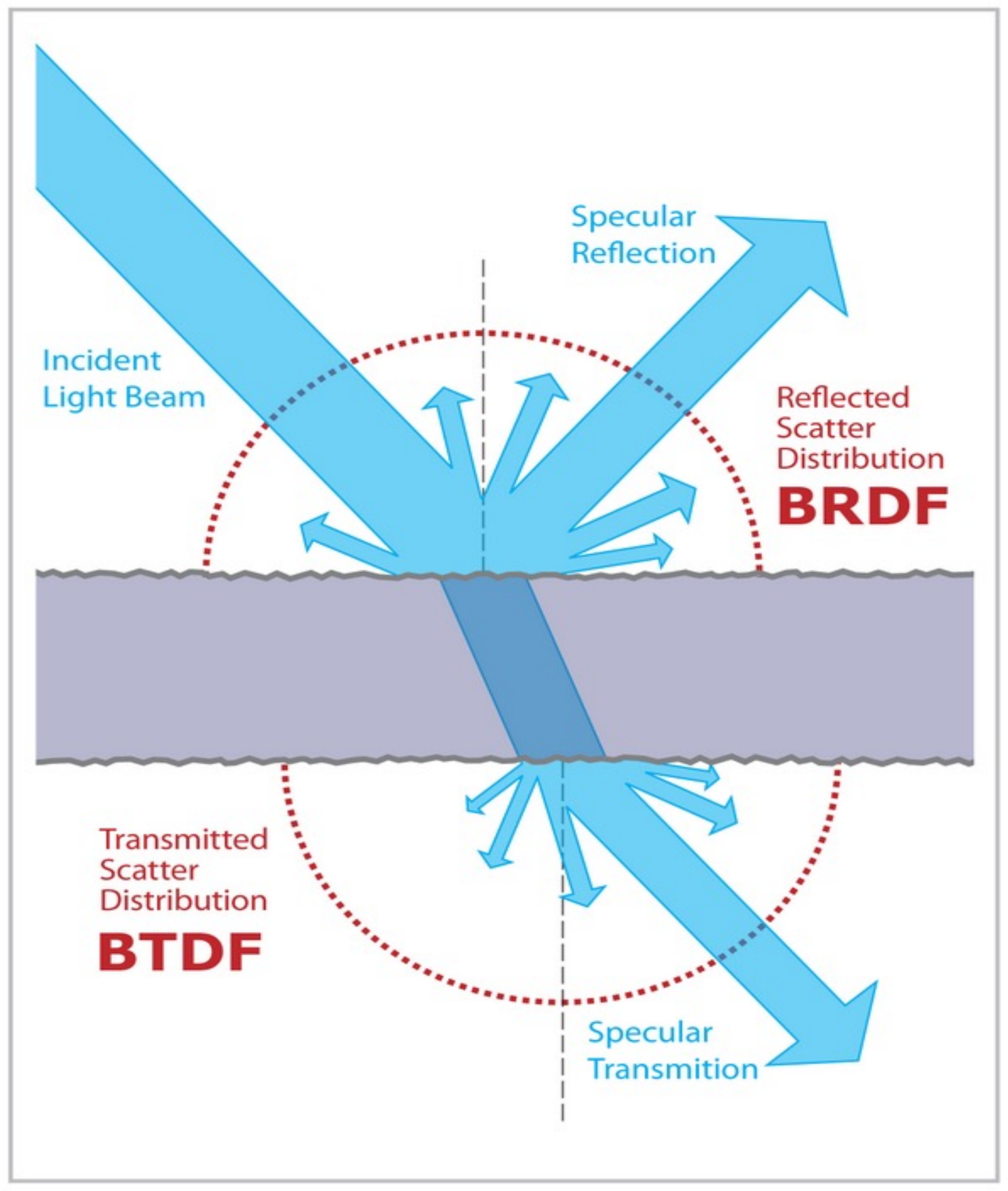
Light/Object Interactions

- When light hits a material, it may be: absorbed, reflected, transmitted
- When light passes through a material, it may be: absorbed, scattered
- When exits a material, it may be: absorbed, reflected, transmitted

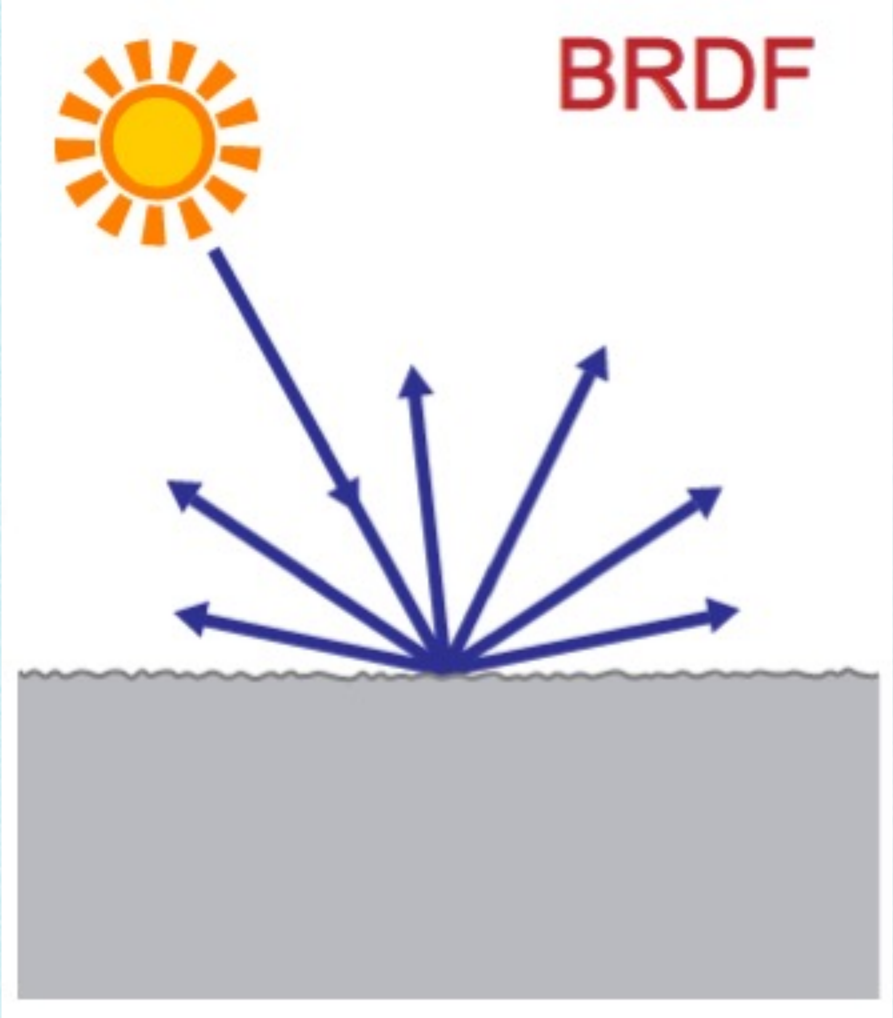


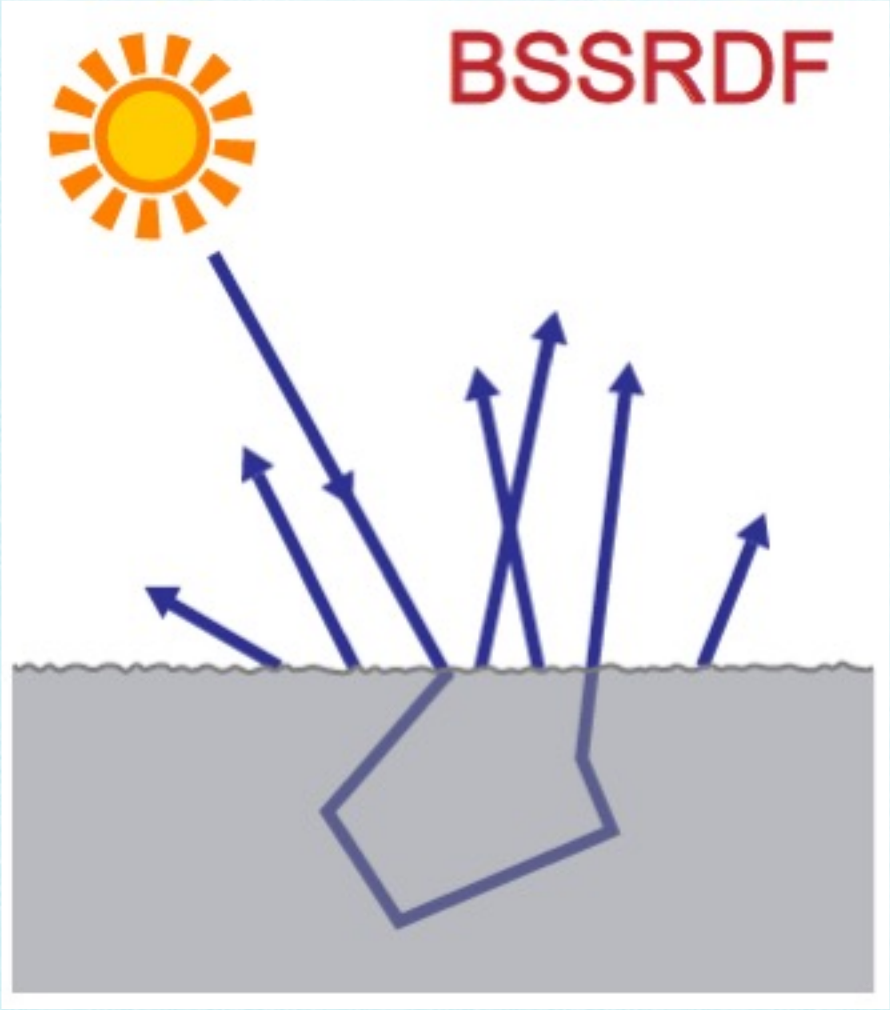
Engineering Approximations

- <u>BRDF</u>
  - Bidirectional Reflectance Distribution Function
  - models how light is <u>reflected</u>
- <u>BTDF</u>
  - Bidirectional Transmittance Distribution Function
  - models how light is <u>transmitted</u>
- <u>BSSRDF</u>
  - Bidirectional Surface Scattering Reflectance Distribution Function
  - <u>combined</u> reflection/transmission model



Opaque (BRDF) vs. Translucent (BSSRDF)





Opaque (BRDF) vs. Translucent (BSSRDF)

Opaque (BRDF) vs. Translucent (BSSRDF)

Opaque (BRDF) vs. Translucent (BSSRDF)

BRDF

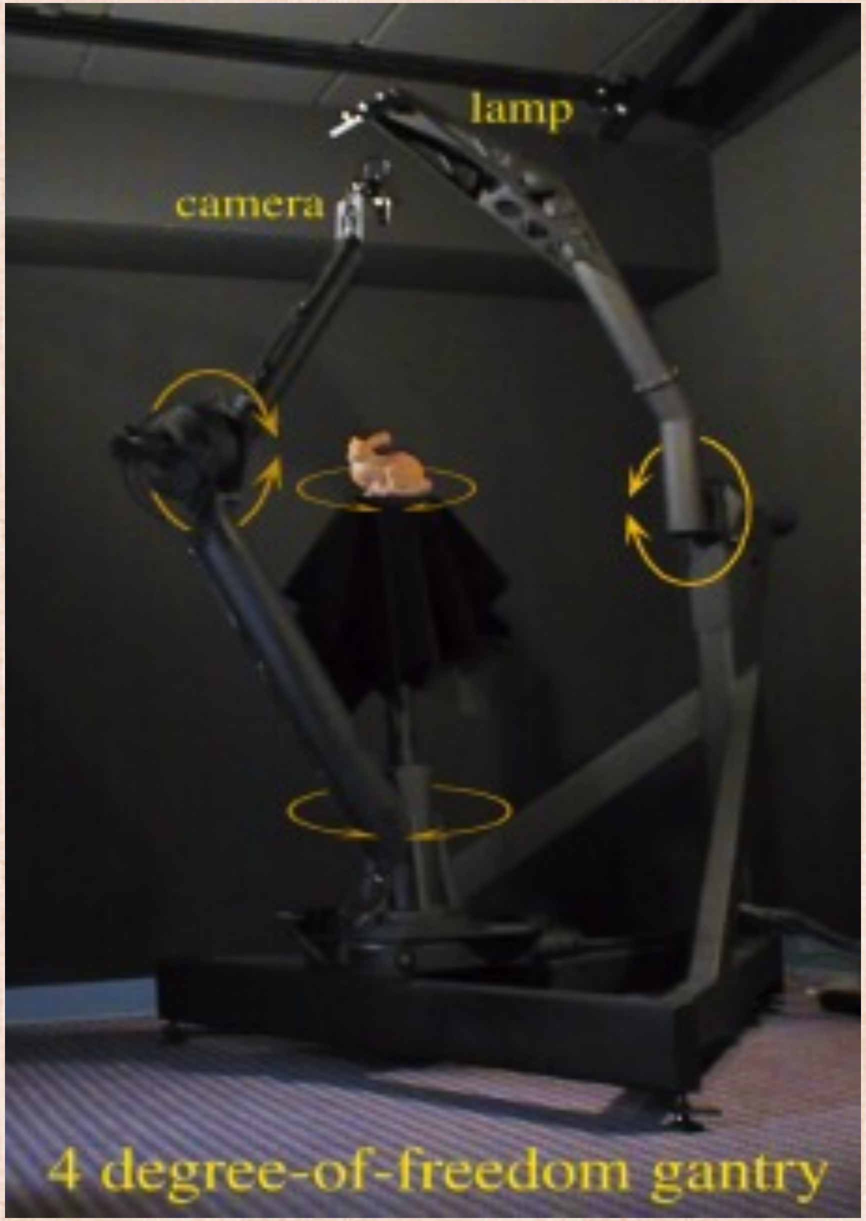
- $BRDF(\lambda, \omega_i, \omega_o, u, v)$
  - λ is the wavelength (we'll cheat with R, G, B as usual)
  - (u, v) are the coordinates on the object's surface (we'll cheat with a texture)
  - $\omega_i(\theta_i, \phi_i)$ and $\omega_o(\theta_o, \phi_o)$ are the incoming/outgoing light directions (parameterized by the 2D surface of a hemisphere)

- Thus, we consider: $BRDF_R(\omega_i, \omega_o)$, $BRDF_G(\omega_i, \omega_o)$, $BRDF_B(\omega_i, \omega_o)$
- These are each 4D functions, i.e. functions of 4 variables $\theta_i, \phi_i, \theta_o, \phi_o$

- Specifically: $BRDF(\omega_i, \omega_o) = \frac{dL_o(\omega_o)}{dE_i(\omega_i)}$
- The outgoing light emitted from a surface patch (acting as an area light), as a fraction of the incoming light hitting that surface patch (irradiance)

Measuring/Approximating a BRDF

- Can measure 4D BRDF data with a gonioreflectometer (to obtain a 4D table of values)
- Alternatively, there are analytical models:
  - Blinn-Phong Model – simplest and general purpose (plastic)
  - Cook-Torrance Model – better specular (metal)
  - Ward Model – anisotropic (brushed metal, hair)
  - Oren-Nayar Model – non-Lambertian (concrete, plaster, the moon)
  - Etc.



The Lighting Equation

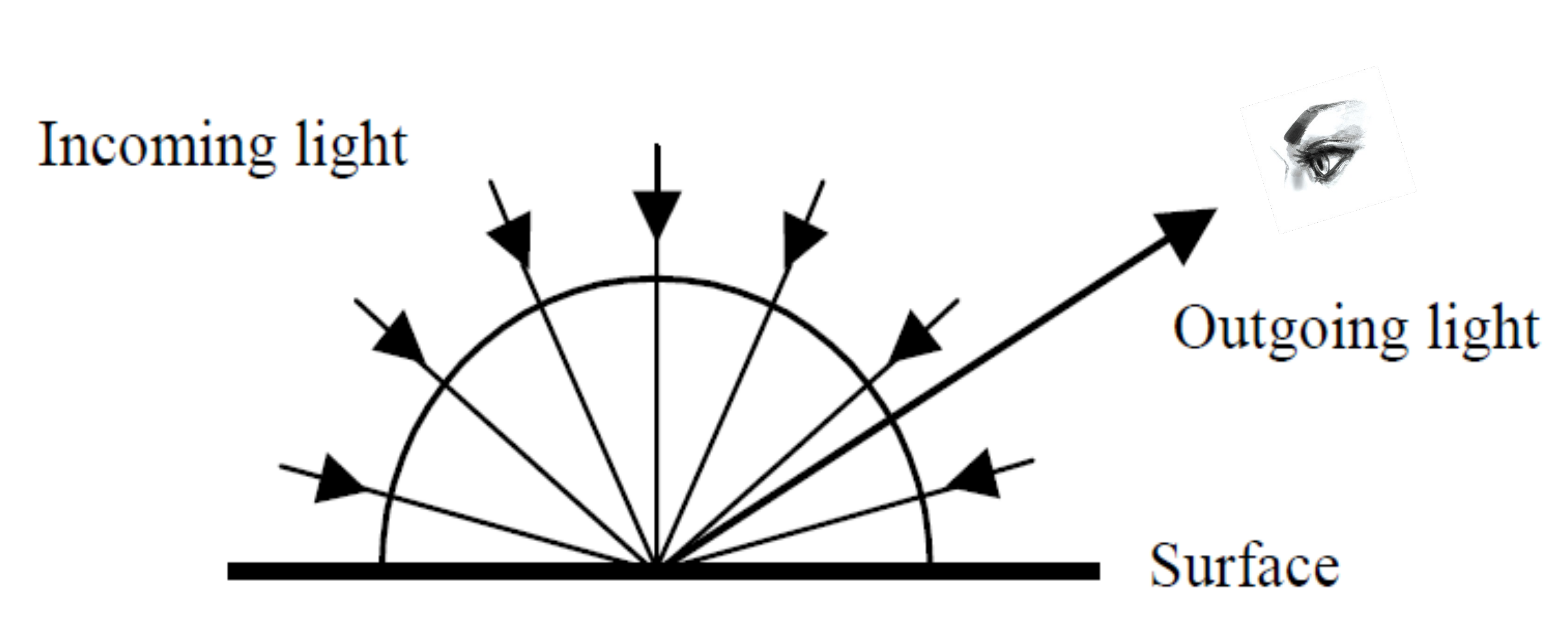
- Given a point on an object:
  - Light from every incoming direction ω_i hits that point
  - For each incoming direction ω_i, light is reflected outwards in every direction ω_o
  - The BRDF indicates what fraction of the light from an incoming direction ω_i is reflected in each of the outgoing directions ω_o

- Light is reflected in all outgoing directions (allowing us all to see the same spot on an object)
- But, we all see different light (so it can, and often does, look differently to each one of us)
- To render a synthetic scene, one (merely) needs to figure out what light each pixel of the camera's film sees

It’s an Integral

- The total amount of light reflected in a single outgoing direction is the sum of the of the light reflected in that direction due to light incoming from every direction:

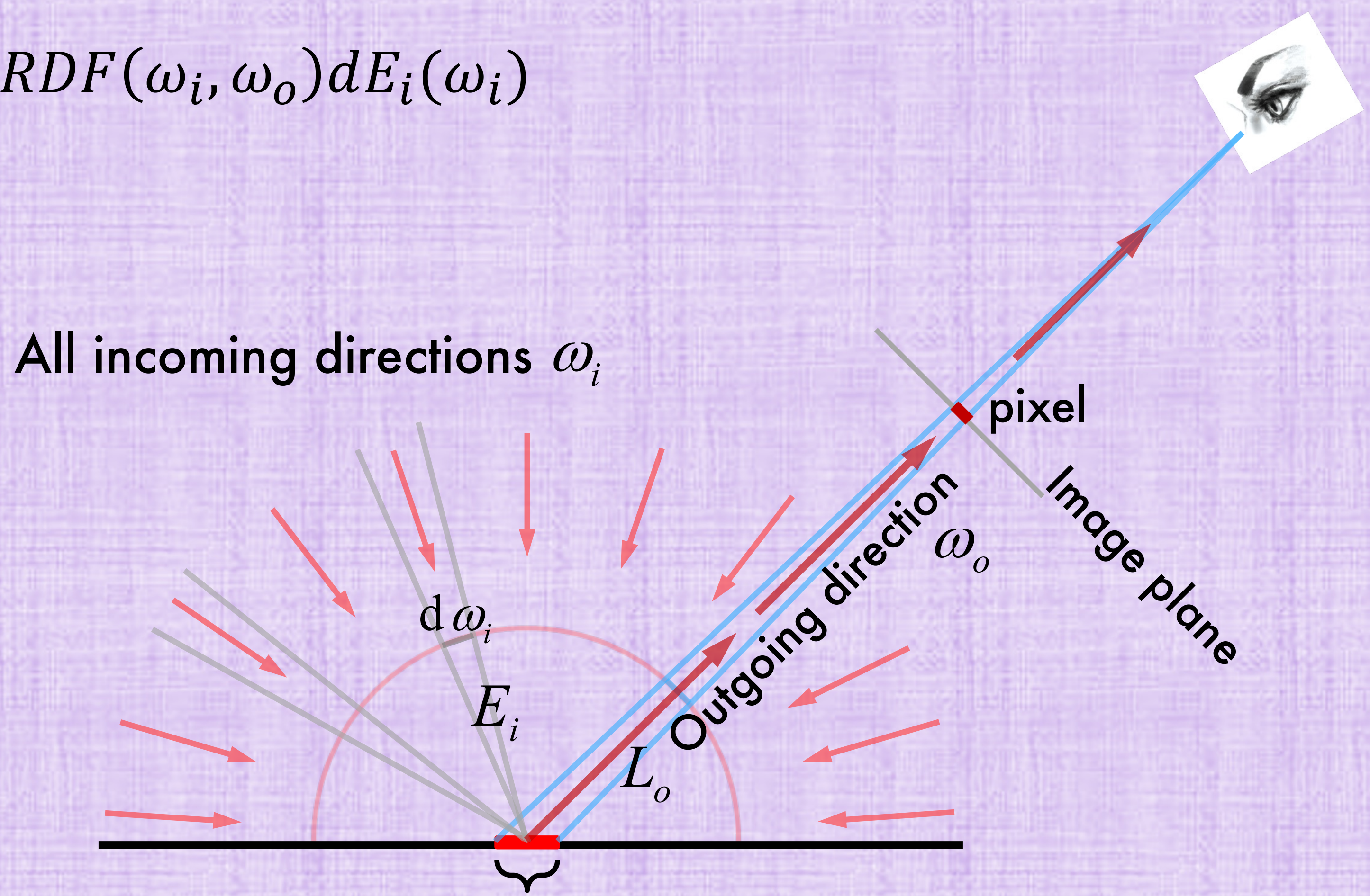
$L_o(\omega_o) = \sum_{i \in in} L_{o\ due\ to\ i}(\omega_i, \omega_o)$



The Lighting Equation

- For each pixel, integrate the BRDF across all incoming directions for every point in the pixel's un-projected area (which acts as an area light)

$$L_o(\omega_o) = \int_{i \in in} BRDF(\omega_i, \omega_o) dE_i(\omega_i)$$



(Radiance only) Lighting Equation

- Multiplying the BRDF by an incoming irradiance gives the outgoing radiance

$$dL_{o\ due\ to\ i}(\omega_i, \omega_o) = BRDF(\omega_i, \omega_o) dE_i(\omega_i)$$

- For even more realistic lighting, we'll bounce light all around the scene
- It's tedious to convert between E and L, so use $dE = Ld\omega \cos\theta$ to obtain:

$$dL_{o\ due\ to\ i}(\omega_i, \omega_o) = BRDF(\omega_i, \omega_o) L_i d\omega_i \cos\theta_i$$

- Then,

$$L_o(\omega_o) = \int_{i \in in} BRDF(\omega_i, \omega_o) L_i \cos\theta_i \, d\omega_i$$

Pixel Color

- Power per unit area hitting a pixel (irradiance):

$$E_i = \int L_i cos\theta_i \, d\omega_i$$

obtained from integrating
$dE = L d\omega cos\theta$

- Assume L and θ are constant across the (very) small pixels:

$$E_{pixel} \approx L_{pixel,ave} \cos\theta_{pixel,ave} \int d\omega_i = (L_{pixel,ave} \cos\theta_{pixel,ave})\omega_{pixel}$$

- If the film is small, $\cos\theta_{pixel,ave} \approx 1$ and $\omega_{pixel} \approx \frac{\omega_{film}}{\# \, pixels}$; then,

$$E_{pixel} \approx \left(\frac{\omega_{film}}{\# \, pixels}\right) L_{pixel,ave}$$

- Thus, can store L instead of E (and scale by constant later)