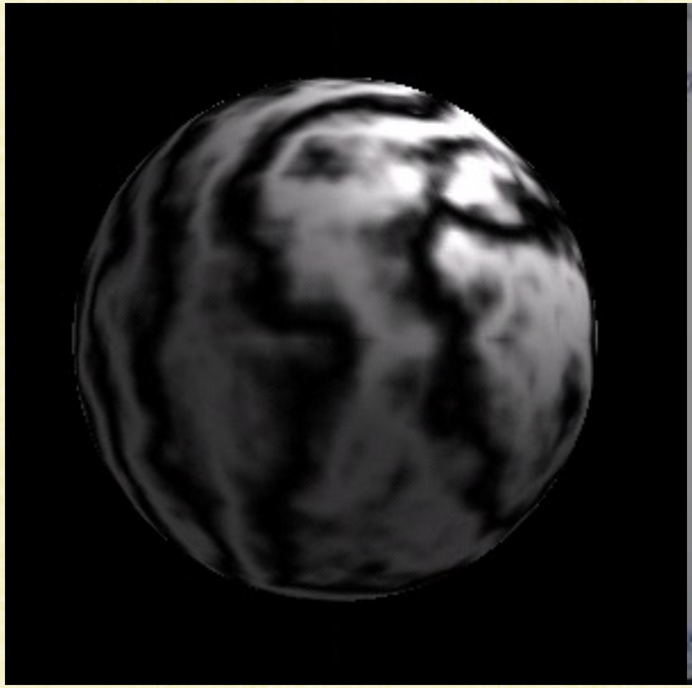
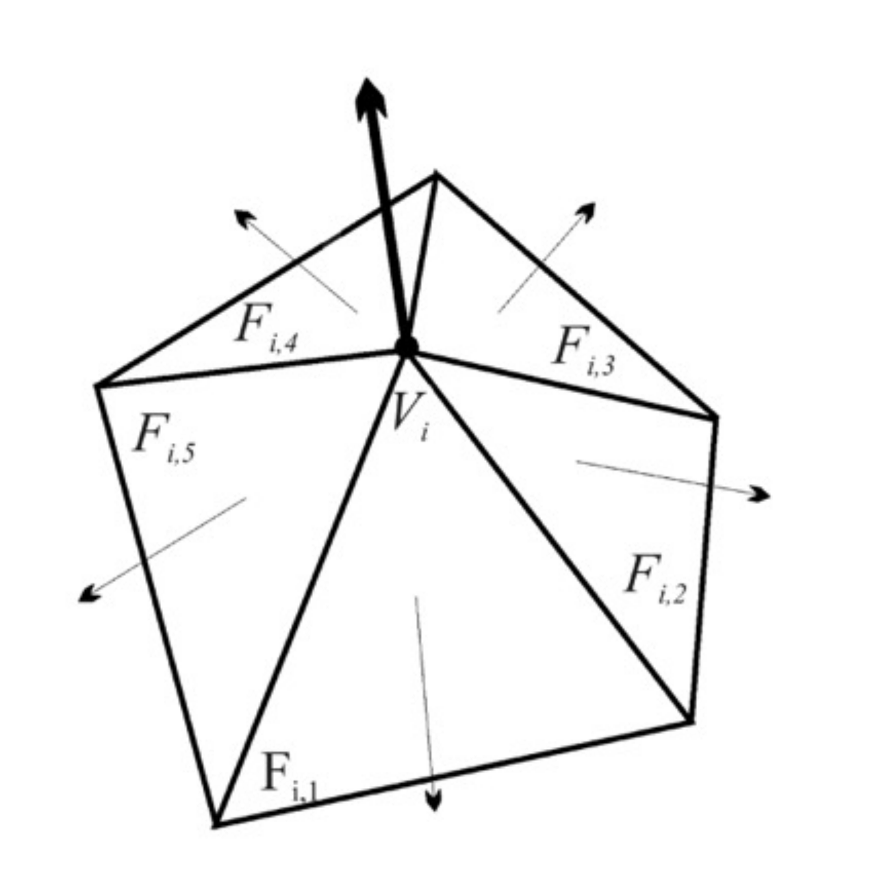
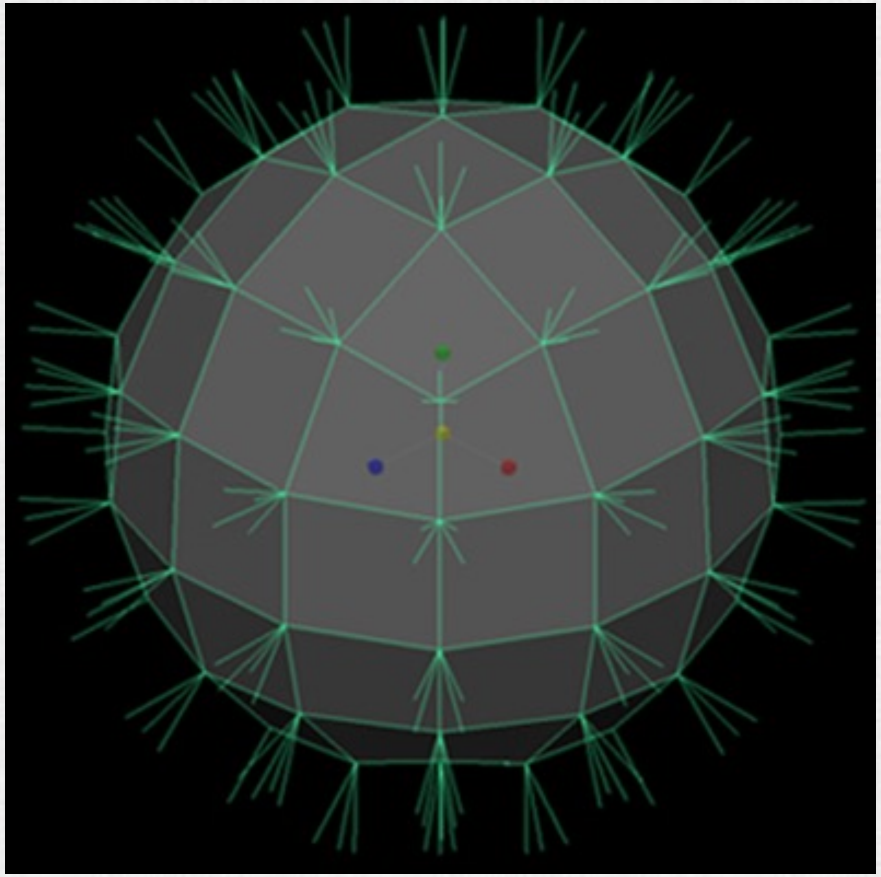
More Texture Mapping



Recall: (Averaged) Vertex Normals

- Each vertex belongs to a number of triangles, each with their own normal
- Averaging those normals (weighted averaging, based on: area, angle, etc.) gives a unique normal for each vertex

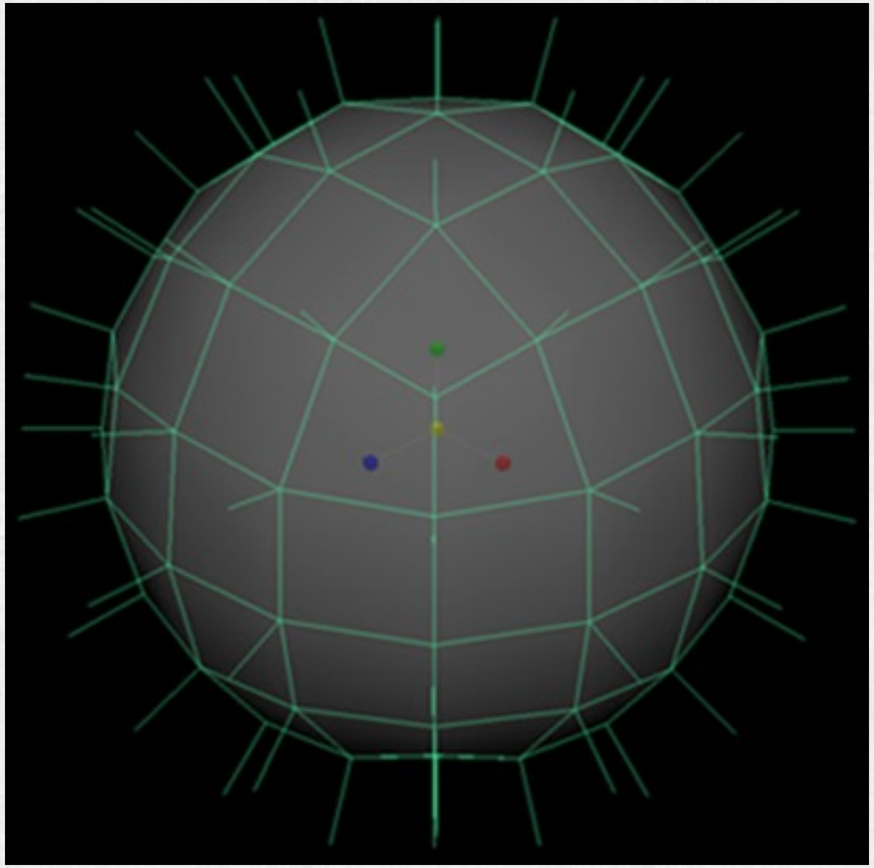




Recall: Smooth Shading

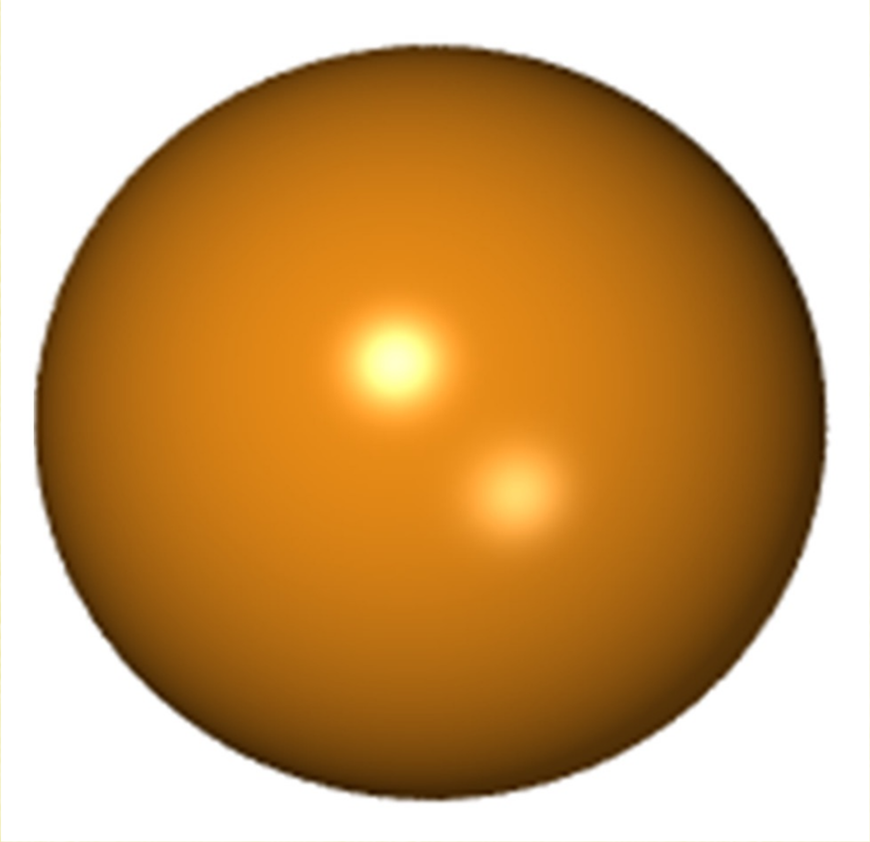
- Use barycentric weights to interpolate (averaged) vertex normals to the interior of the triangle:

$$\hat{N}_p = \frac{\alpha_0\hat{N}_0 + \alpha_1\hat{N}_1 + \alpha_2\hat{N}_2}{\left\|\alpha_0\hat{N}_0 + \alpha_1\hat{N}_1 + \alpha_2\hat{N}_2\right\|_2}$$

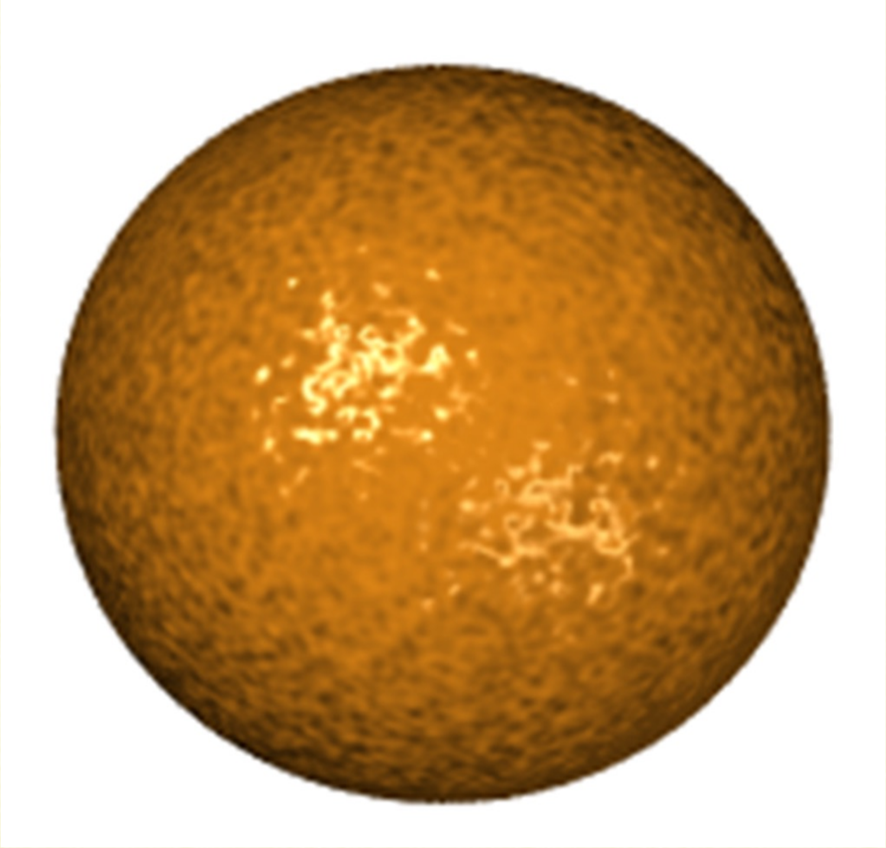


Perturbing the Normal

- Store a normal vector in the texture (instead of a color)
- This perturbed normal can “fake” geometric details



using real normal



using fake normal

Bump Map

- Single-channel (grey-scale) height map h_{ij}, representing the height at location (u_i, v_j)
- The tangent plane at a point (u_i, v_j, h_{ij}) is: $-\frac{\partial h(u_i,v_j)}{\partial u}(u-u_i)-\frac{\partial h(u_i,v_j)}{\partial v}(v-v_j)+(h-h_{ij})=0$
- So, the outward (non-unit) normal is: $\left(-\frac{\partial h(u_i,v_j)}{\partial u},-\frac{\partial h(u_i,v_j)}{\partial v},1\right)$
- Partial derivatives are computed via finite differences: $\frac{\partial h(u_i,v_j)}{\partial u}=\frac{h_{i+1,j}-h_{i-1,j}}{u_{i+1}-u_{i-1}}$ and $\frac{\partial h(u_i,v_j)}{\partial v}=\frac{h_{i,j+1}-h_{i,j-1}}{v_{j+1}-v_{j-1}}$

Normal Map

- A normalized vector has each component in $[-1,1]$, so one can convert back and forth to a color via:

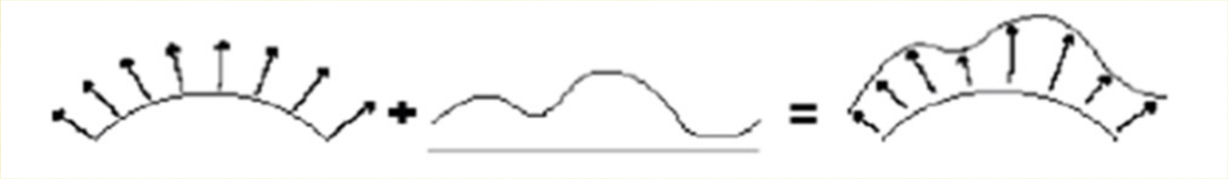
$$(R,G,B) = 255\frac{\vec{N}+(1,1,1)}{2} \quad \text{and} \quad \vec{N} = \frac{2}{255}(R,G,B) - (1,1,1)$$

- Normal maps use more storage than bump maps, but require less computation

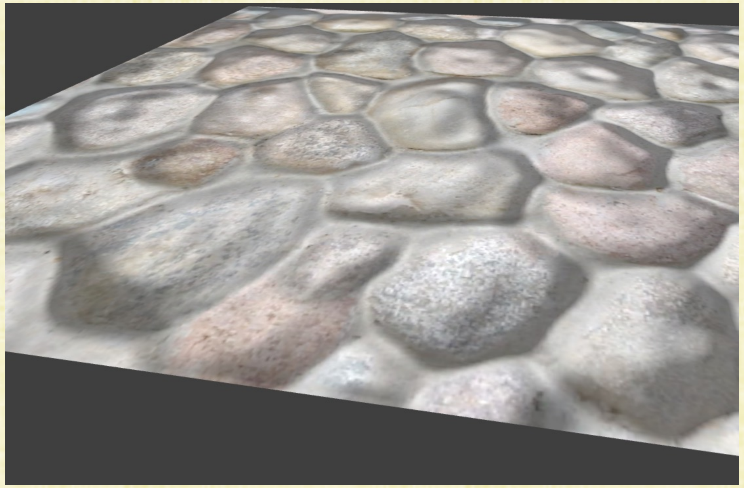
normal mapping on a plane
(note the variation in specular highlights created by variation of the normal)

Displacement Mapping

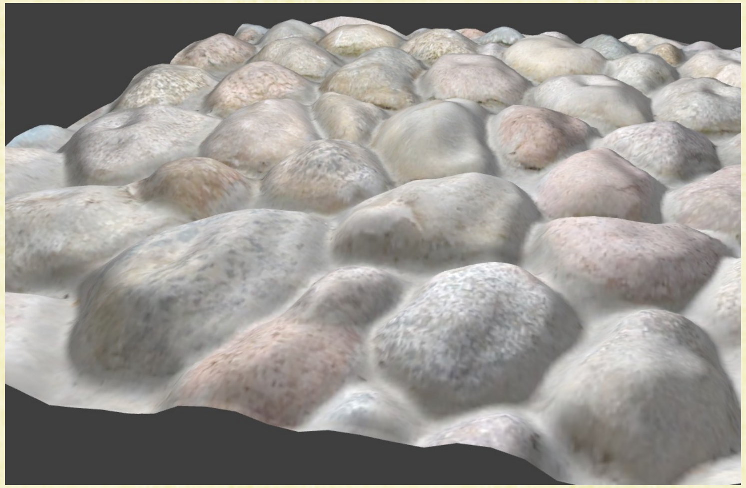
- Subdivide geometry at render time, and use a height map $h(u, v)$ to perturb vertices in the normal direction
- Pros: self-occlusion, self-shadowing, correct silhouettes
- Cons: expensive, requires adaptive tessellation, still need bump/normal map for sub-triangle detail



original geometry $\boldsymbol{h(u,v)}$ displacement map



bump map



displacement map

Displacement Mapping

bump map

displacement map

Recall: Measuring Incoming Light

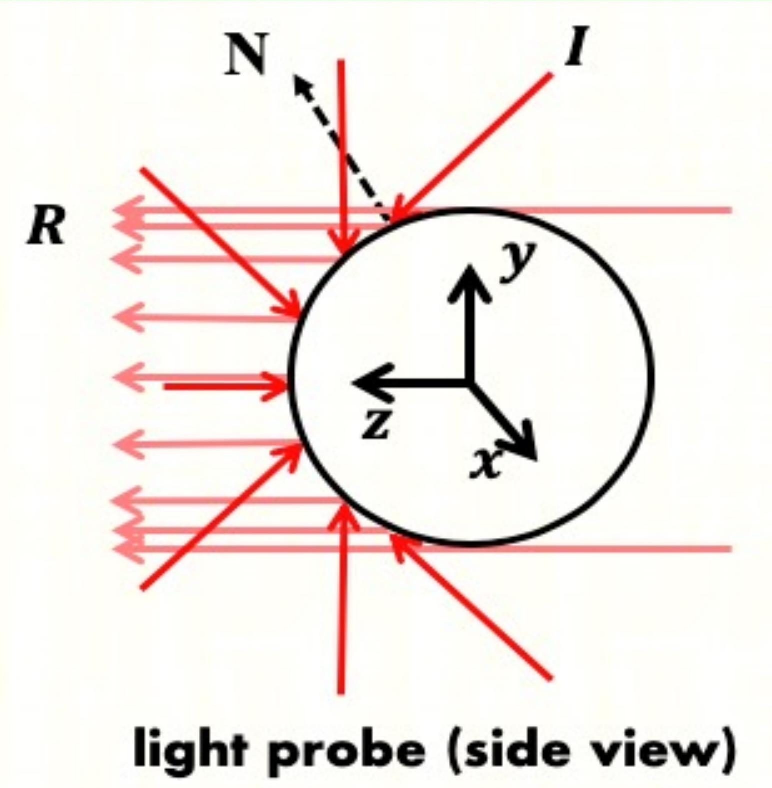
- <u>Light Probe</u>: a small reflective chrome sphere
- Photograph it, in order to record the incoming light (at its location) from all directions

Recall: Using the (measured) Incoming Light

- The (measured) incoming light can be used to render a synthetic object (with realistic lighting)

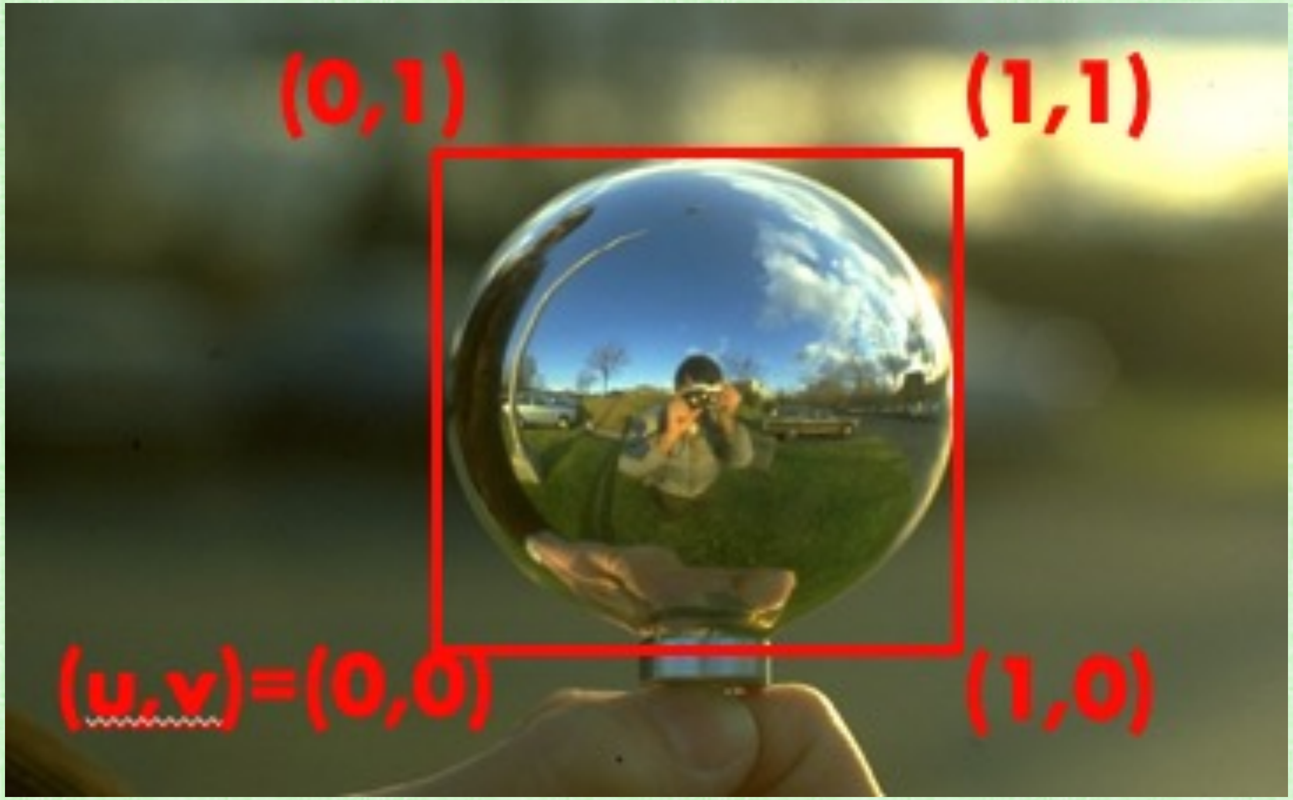
Environment Mapping

- Place a coordinate system at the center of the sphere, so the surface normal is: $\boldsymbol{N} = \frac{1}{\sqrt{x^2+y^2+z^2}}(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z})$
- $\boldsymbol{R}$ is the direction from the light probe to the camera
- Since $\boldsymbol{I}$ and $\boldsymbol{R}$ are equal-angle from $\boldsymbol{N}$ (because of mirror reflection), $\boldsymbol{N}$ has a one-to-one correspondence with $\boldsymbol{I}$



Environment Mapping

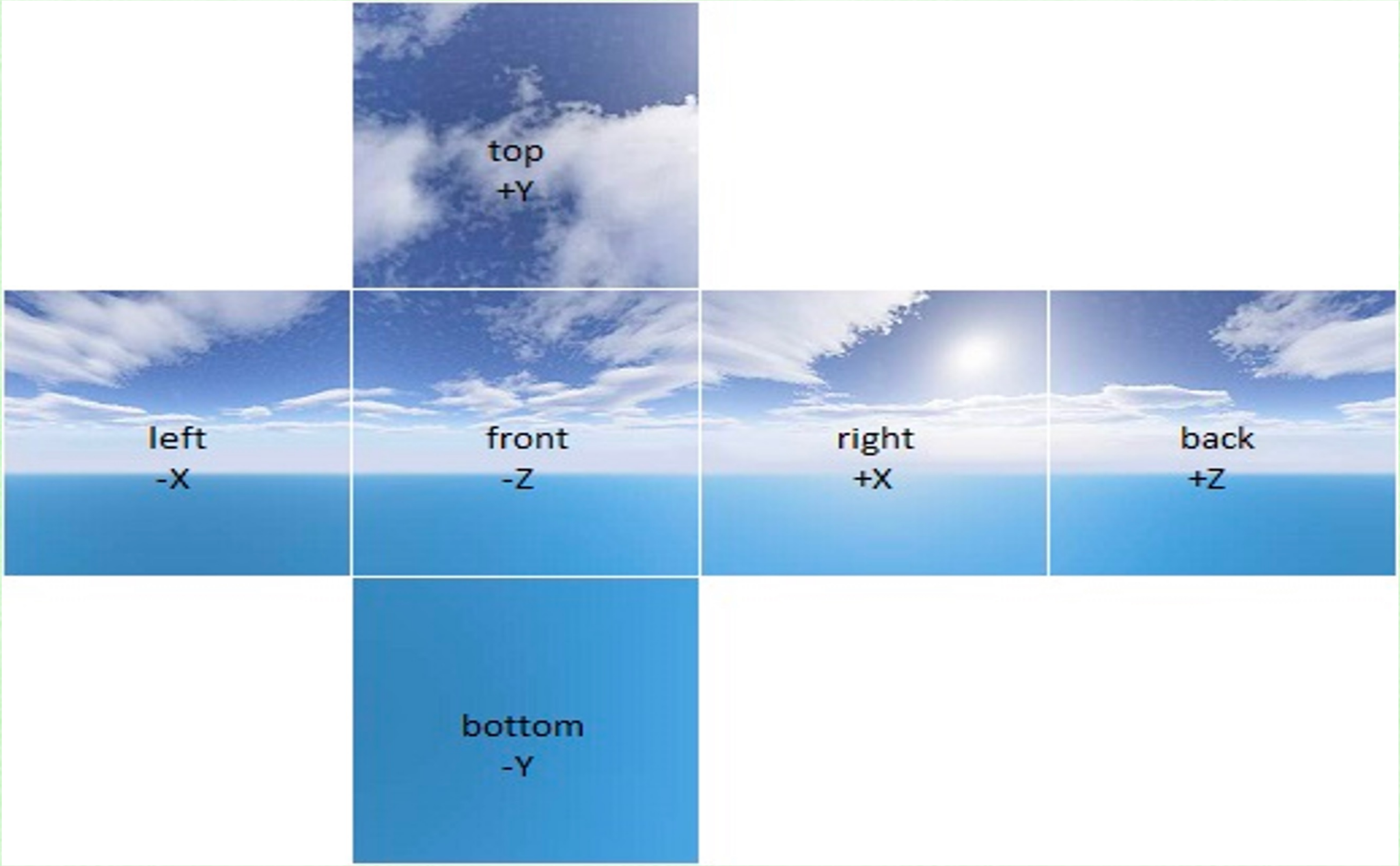
- Given a normal on the geometry being rendered:
- Use $\boldsymbol{n_x}$ and $\boldsymbol{n_y}$ (which are in [-1, 1]) to obtain texture coordinates $(\boldsymbol{u}, \boldsymbol{v}) = \frac{\mathbf{1}}{\mathbf{2}}(\boldsymbol{n_x} + \mathbf{1}, \boldsymbol{n_y} + \mathbf{1})$
- Then, look up the incoming light in the texture (which is a picture of the chrome sphere)



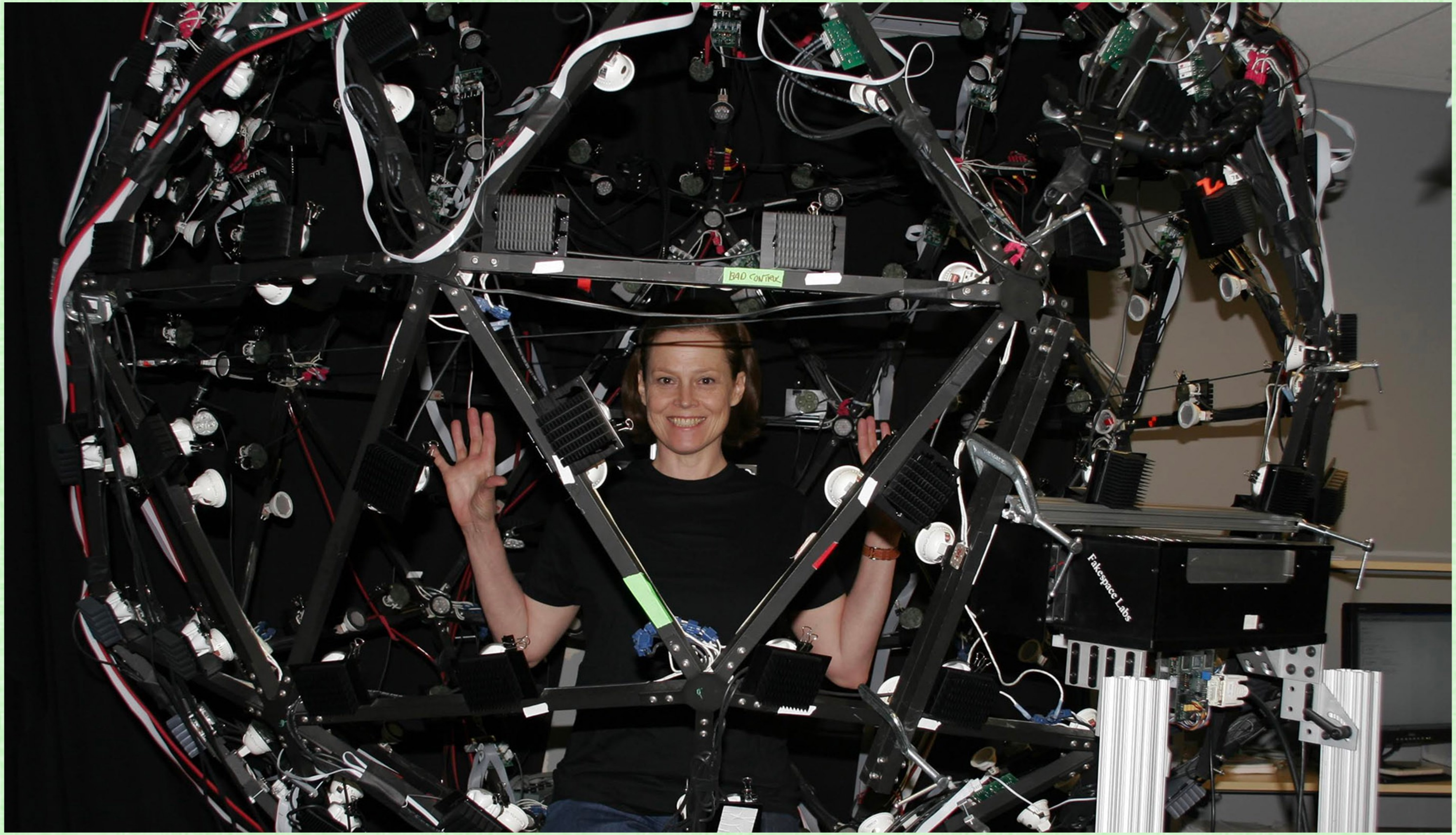
Environment Mapping

Sky Boxes

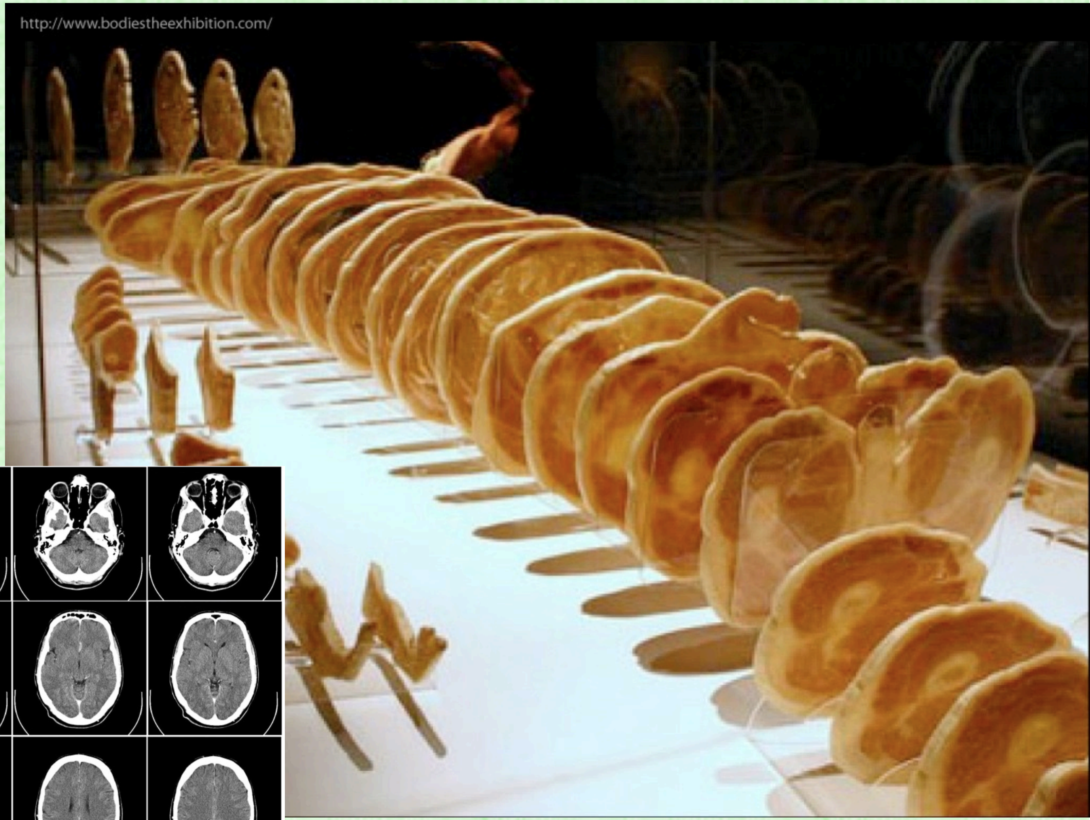
- Model the sky with a texture on the inside of geometry.

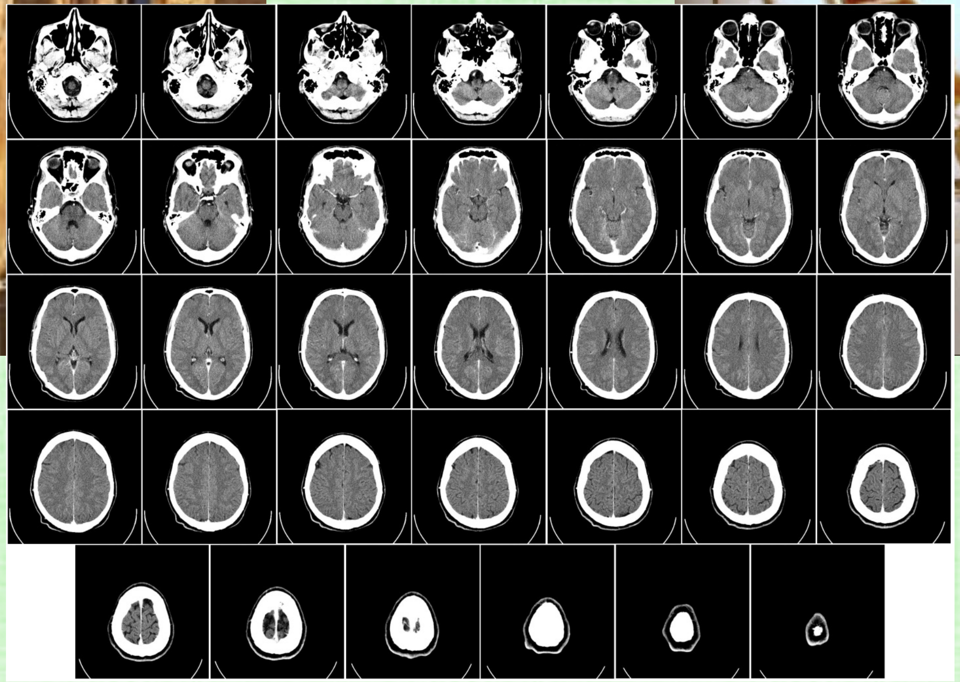


Texture Acquisition via Imaging



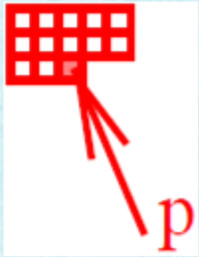
Texture Acquisition via Medical Imaging



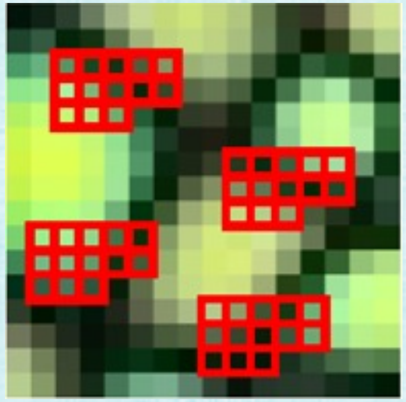


Texture Synthesis: Pixel Based

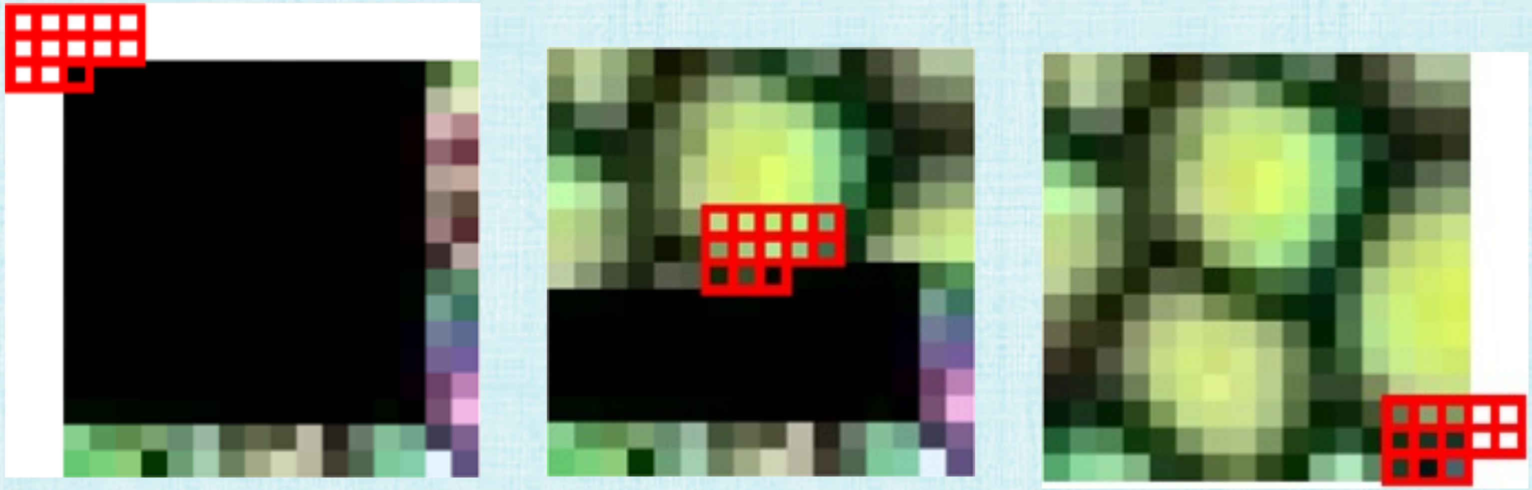
- Create a larger texture (one pixel at a time) from a small sample (using its structural content)
- Generate the texture in a raster scan ordering
- To generate the texture for pixel *p*
  - compare *p*'s neighboring pixels in the (red) stencil to all potential choices in the sample
  - choose the one with the smallest difference to fill pixel p
- When the stencil needs values outside the domain, use periodic boundaries (so, fill the last few rows/columns with random values)



stencil

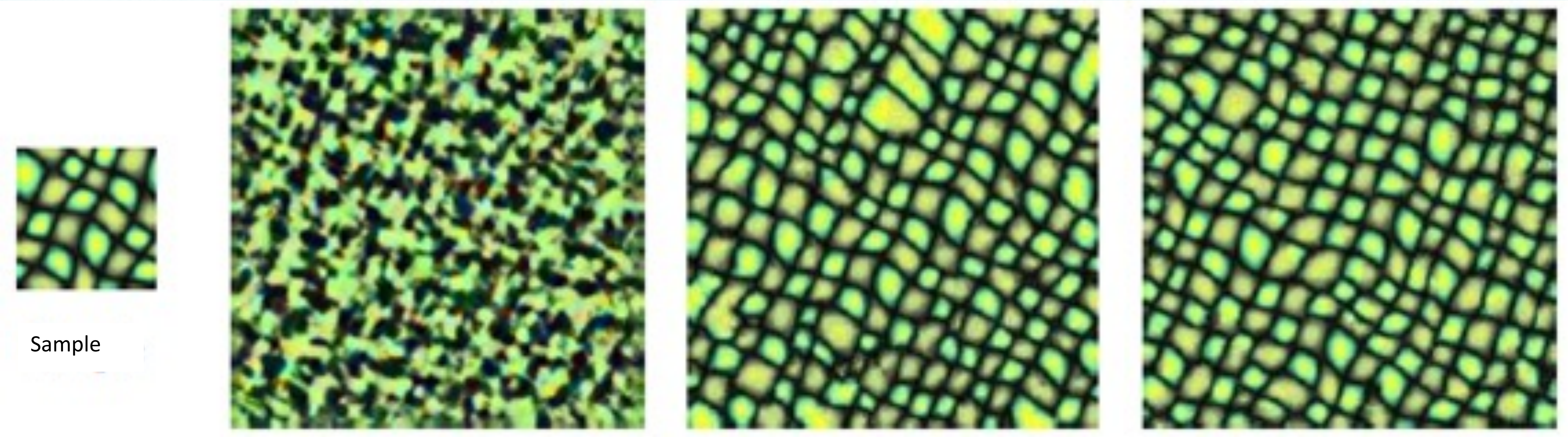


texture sample



raster scan ordering (with randomly generated periodic boundaries)

Texture Synthesis: Pixel Based



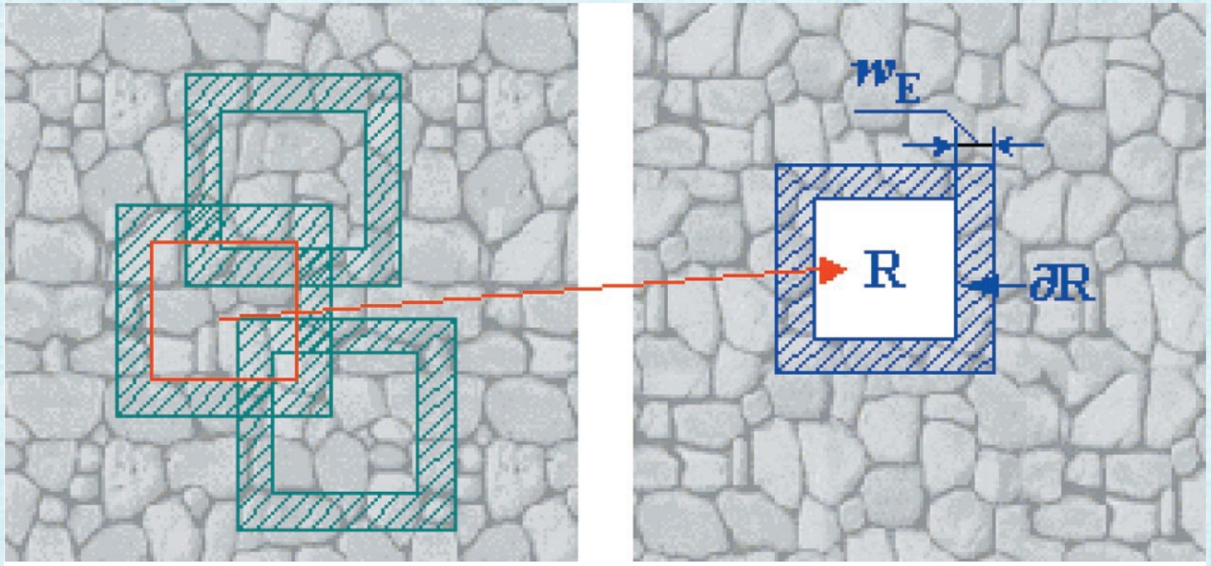
Heeger and Bergen

Efros and Leung

Wei and Levoy

Texture Synthesis: Patch Based

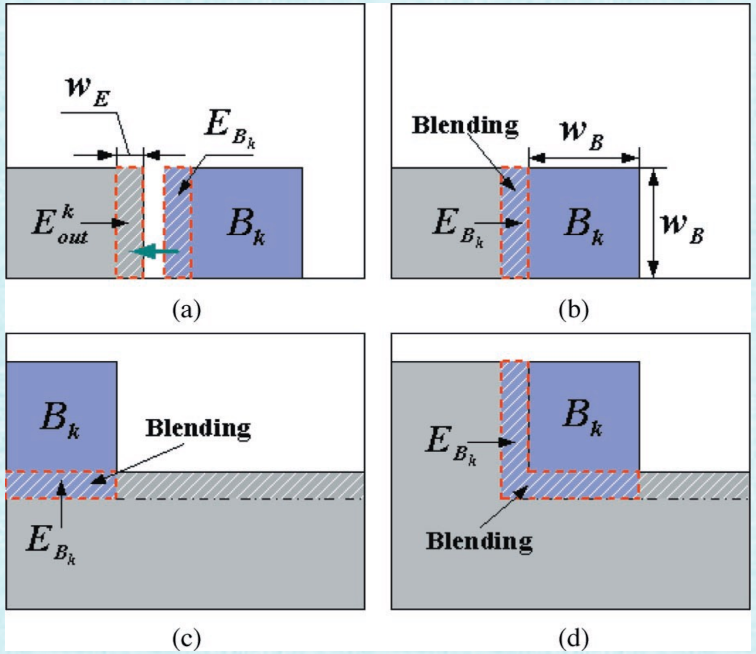
- For each patch:
  - search the original sample to find the candidate that best matches the opverlap boundaries
  - choose the best candidate
  - blend overlapped regions to remove “seams”



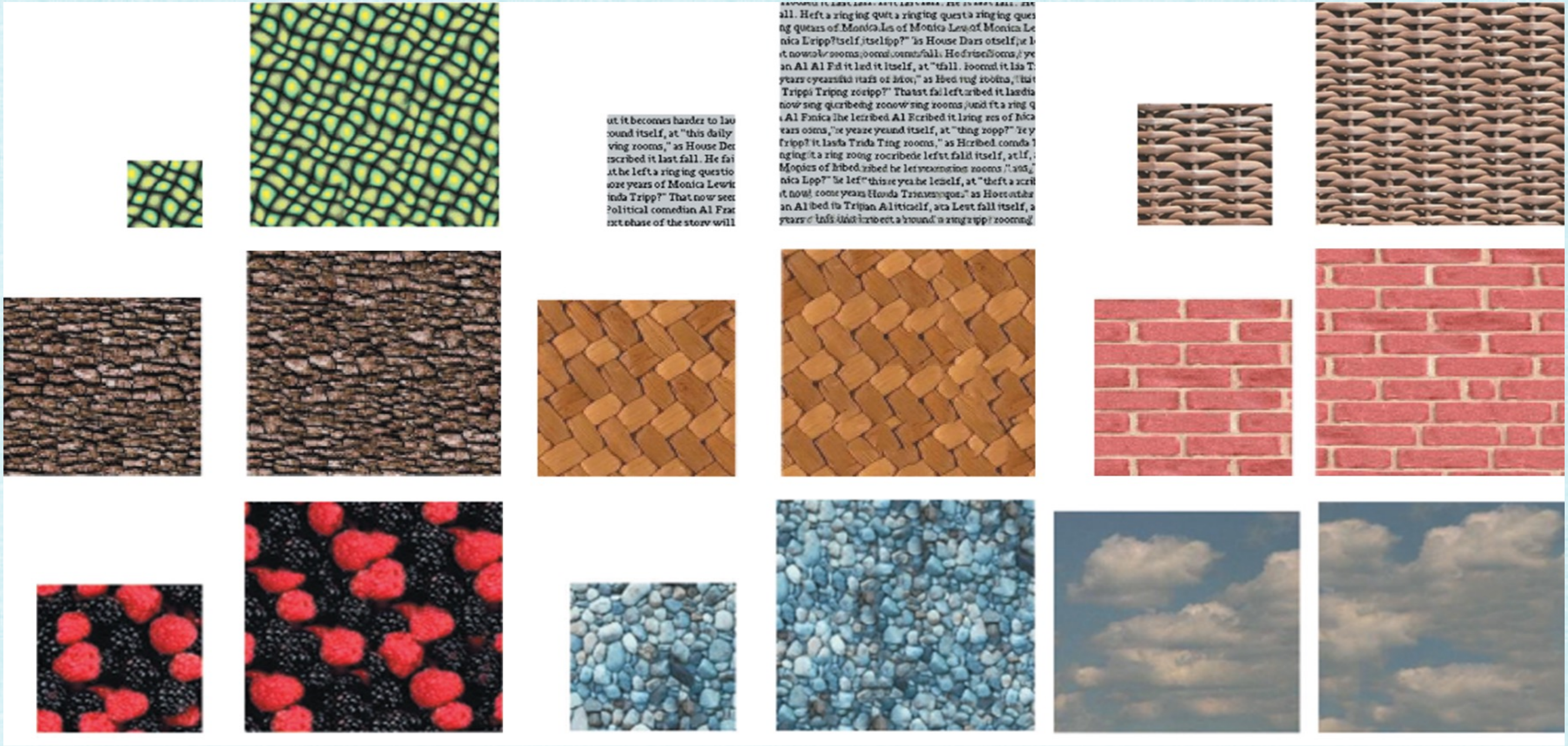
sample

texture

matching boundary regions

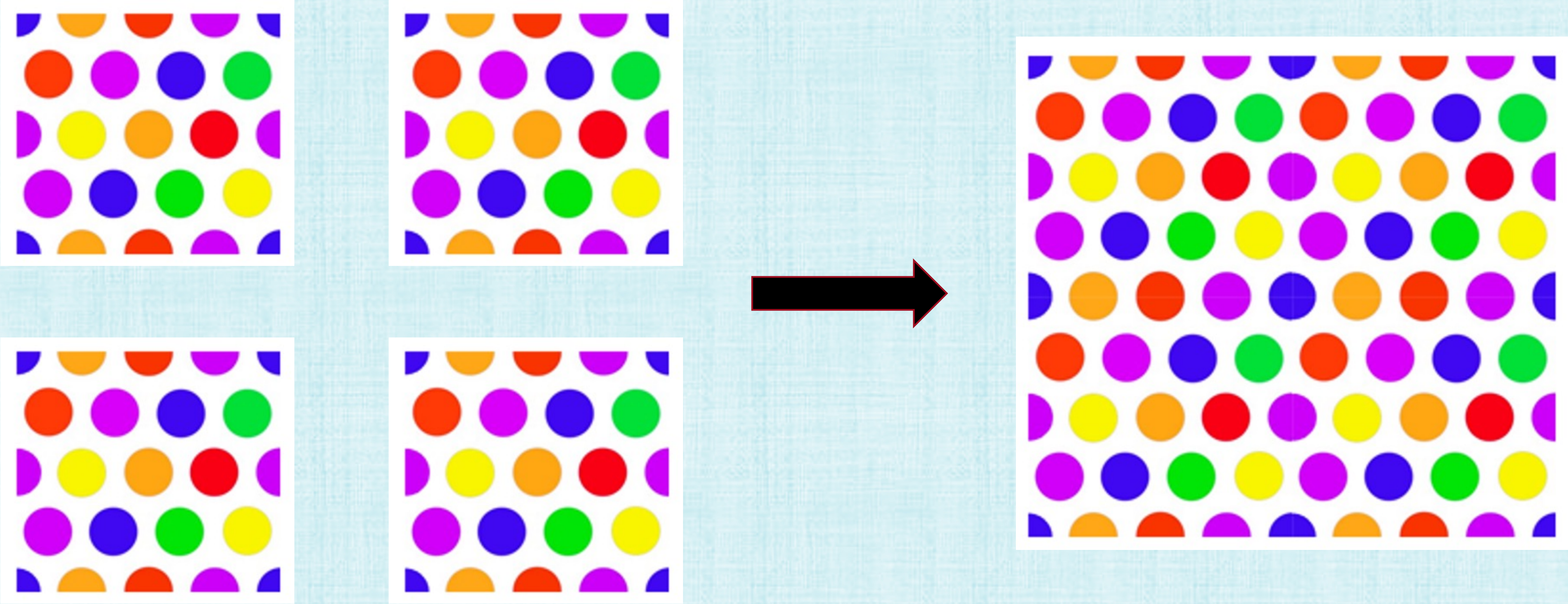


Texture Synthesis: Patch Based



Don’t Stretch Textures!

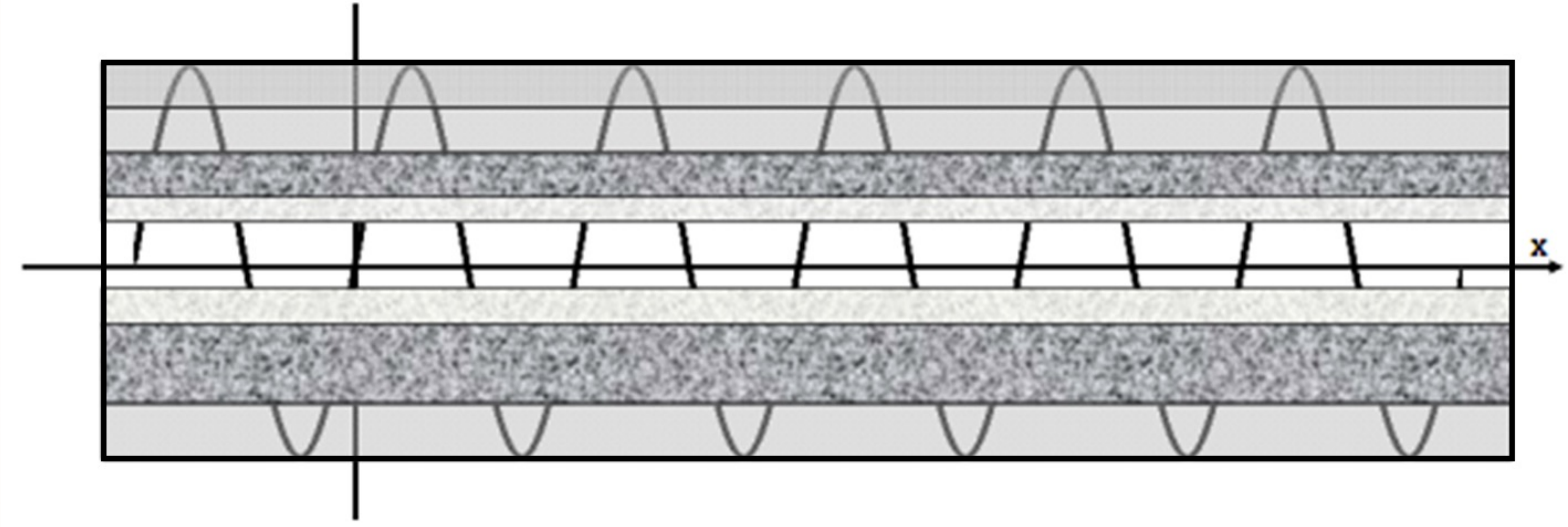
- Stretching out 10 bricks to cover an entire wall of a building is going to look unrealistic!
- Instead, can tile textures if the tiles are made with periodic boundaries



Marble Texture

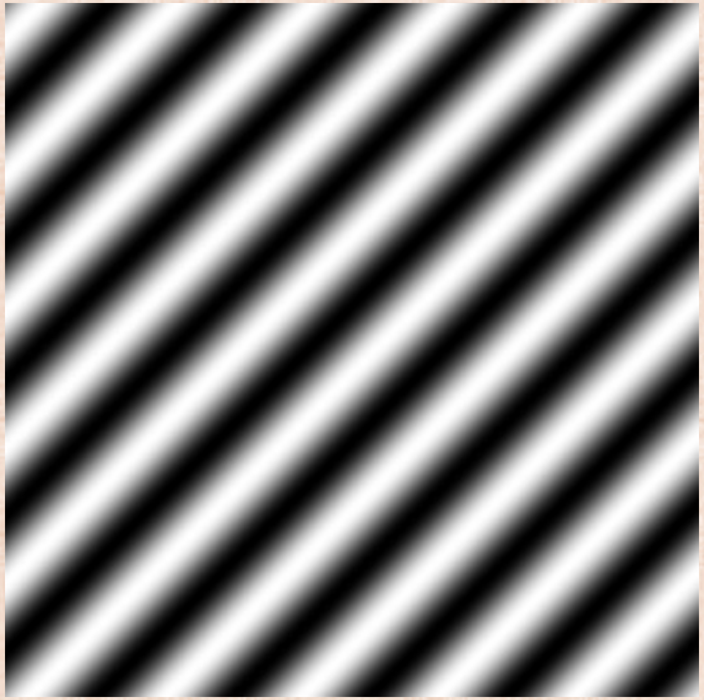
- Define layers of different colors
- Use a function to map layer colors to (u, v) texture locations
- For example:

$$marbleColor(u, v) = LayerColor\big(\sin(k_u u + k_v v)\big)$$

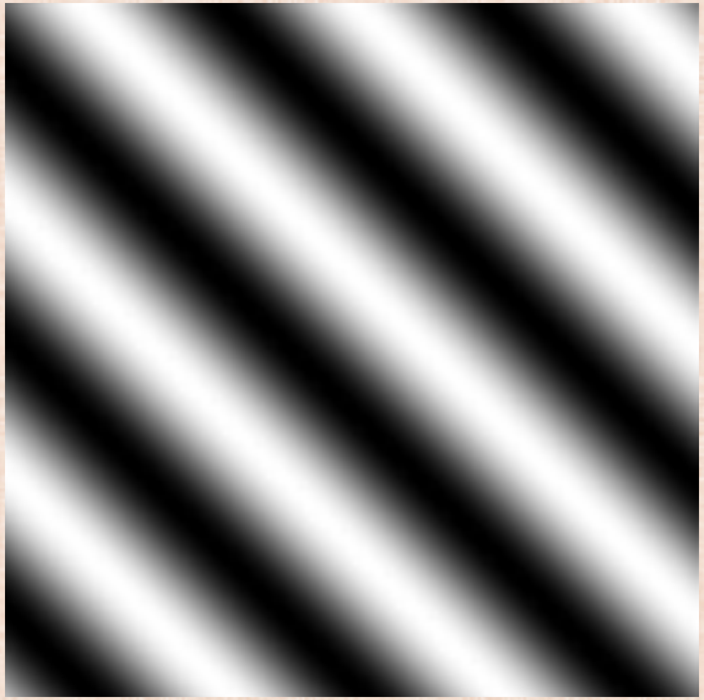


Marble Texture

- k_u and k_v are spatial frequencies
- (k_u, k_v) determines the direction, and $\frac{2\pi}{\sqrt{k_u^2+k_v^2}}$ determines the periodicity
- Problem: too regular (still need to add noise/randomness)



higher frequency



lower frequency

Perlin Noise

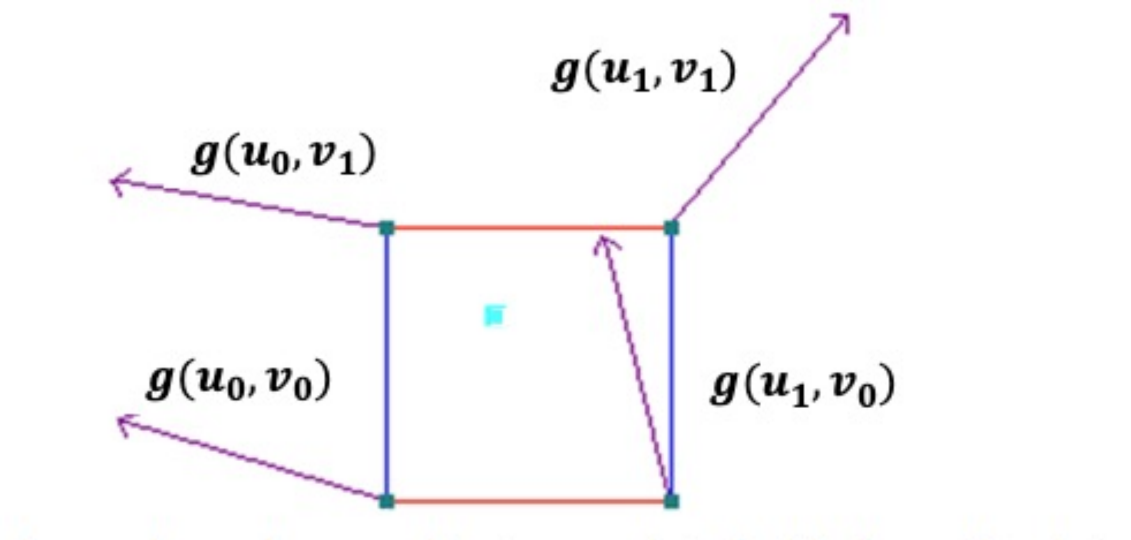
- Noise should have both coherency and structure, in order to look more natural
- Ken Perlin proposed a specific (and amazing!) method for doing this

Perlin Noise

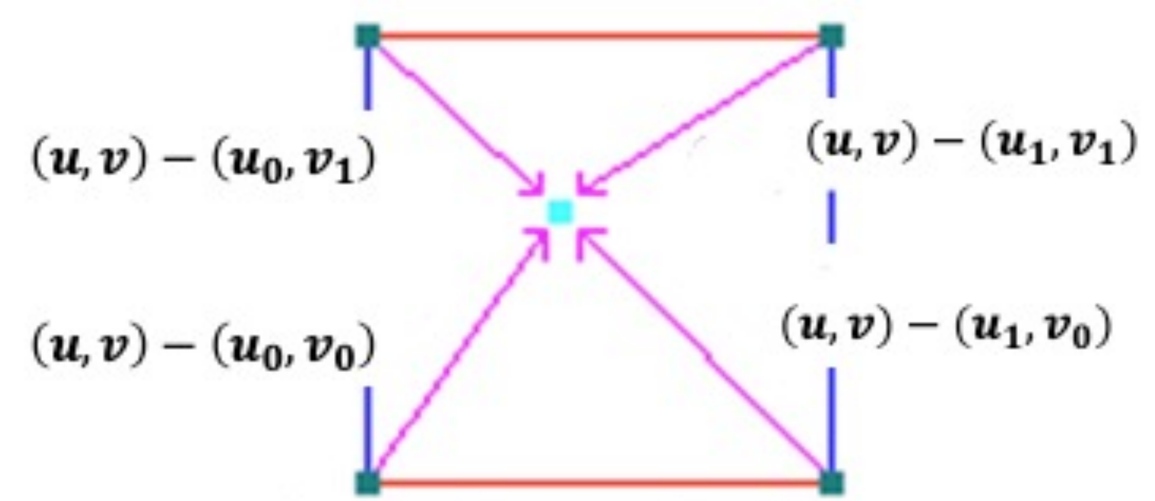
- Place a 2D grid over the texture image, and assign a random (unit) gradient $g(u_i, v_j)$ to each grid point
- For each pixel, compute the dot-products between vectors from the grid corners and the corresponding gradients
- Take a weighted average of the result:

$$noise(u,v) = \sum_{i=0,1;j=0,1} w\left(\frac{u-u_i}{\Delta u}\right) w\left(\frac{v-v_j}{\Delta v}\right) g(u_i, v_j) \cdot (u-u_i, v-v_j)$$

- Cubic weighting: $w(t) = 2|t|^3 - 3|t|^2 + 1$ for $-1 < \mathrm{t} < 1$



4 pseudorandom gradients associated with the grid points



Vectors from the grid points to (u,v)

Multiple Scales

- Natural textures tend to contain a variety of feature sizes
- Mimic this by adding together noises with different frequencies and amplitudes:

$$perlin(u, v) = \sum_k noise\big(frequency(k) * (u, v)\big) * amplitude(k)$$

- Each successive noise function is twice the frequency of the previous one:

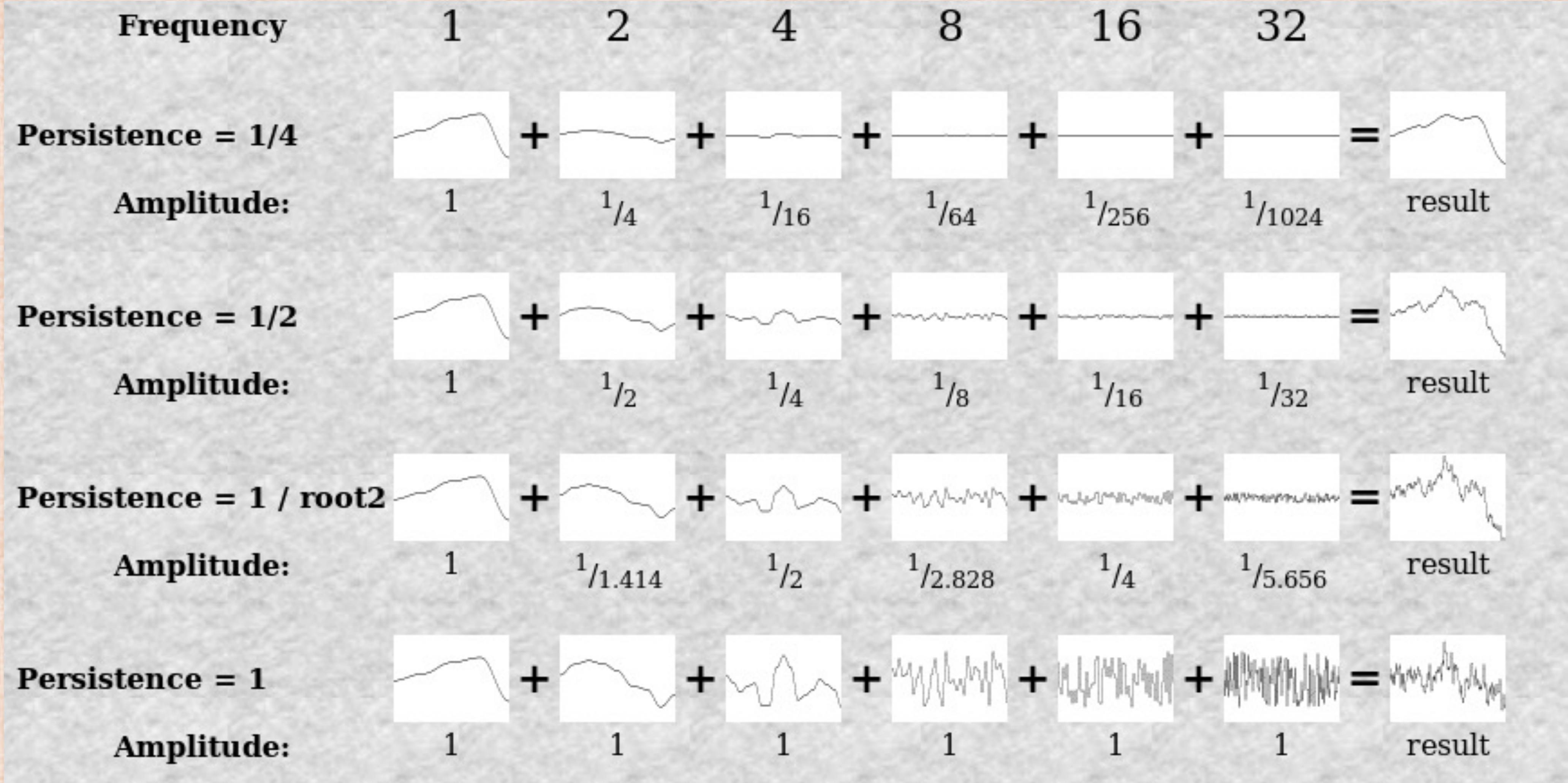
$$frequency(k) = 2^k$$

- The amplitude of higher frequencies is measured by a persistence parameter (≤ 1)
- Thus, higher frequencies have a diminished contribution:

$$amplitude(k) = persistence^k$$

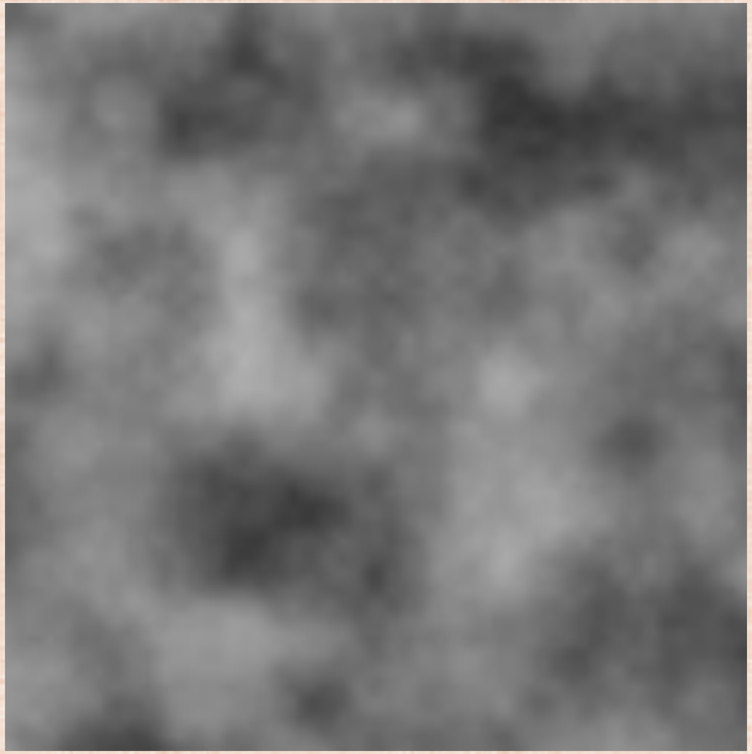
1D Examples

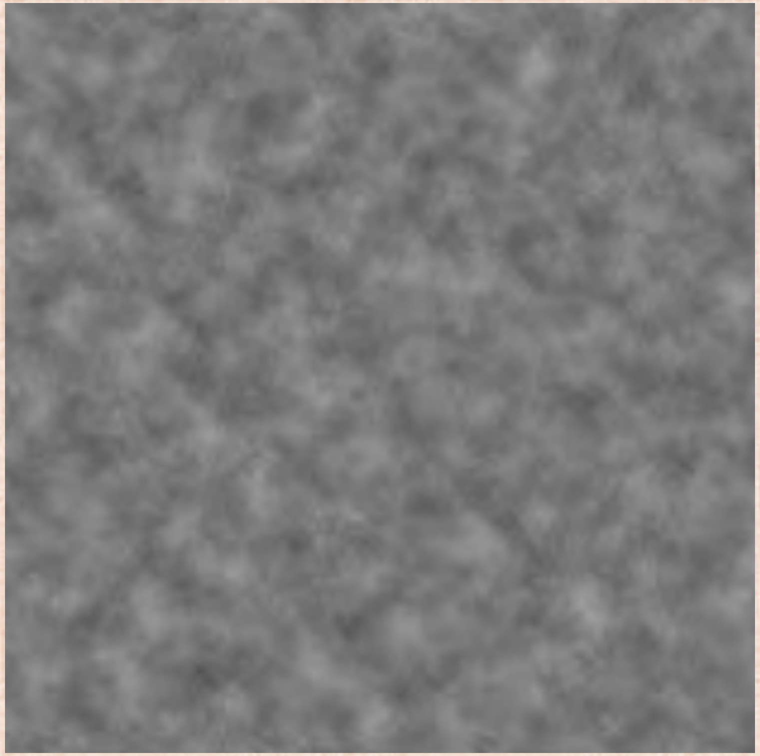
- Smaller persistence gives less higher frequency noise and thus a smoother result

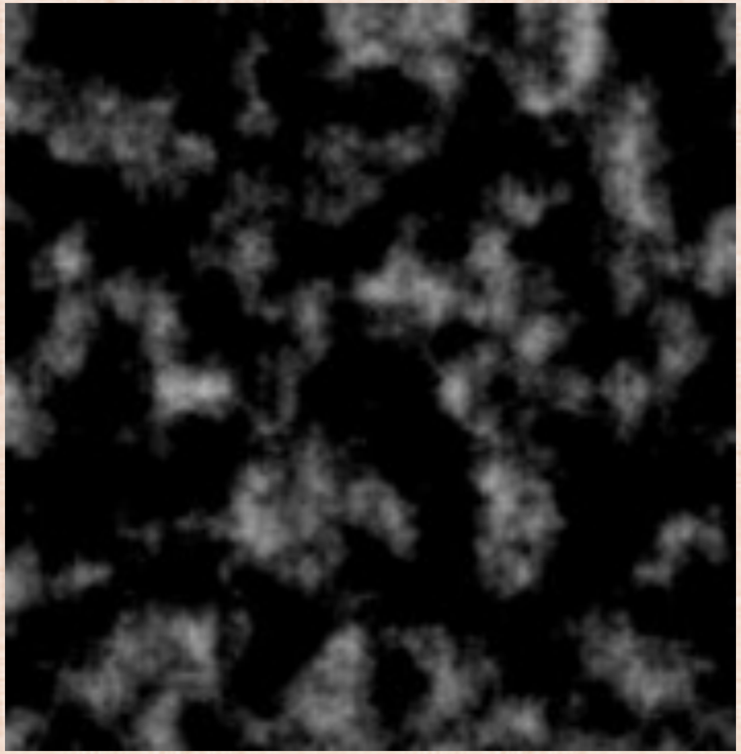


(from: http://freespace.virgin.net/hugo.elias/models/m_perlin.htm)

2D Examples



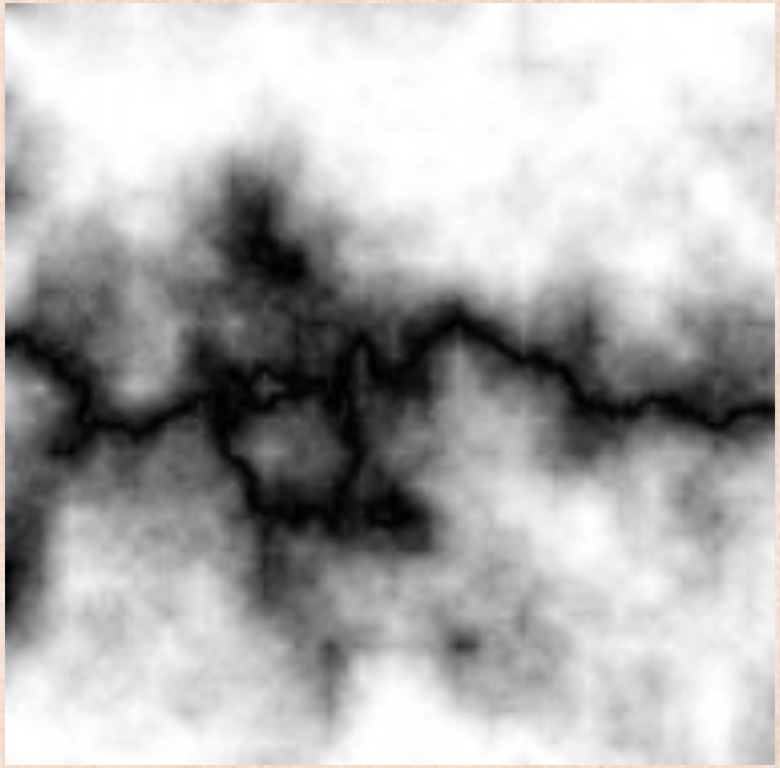


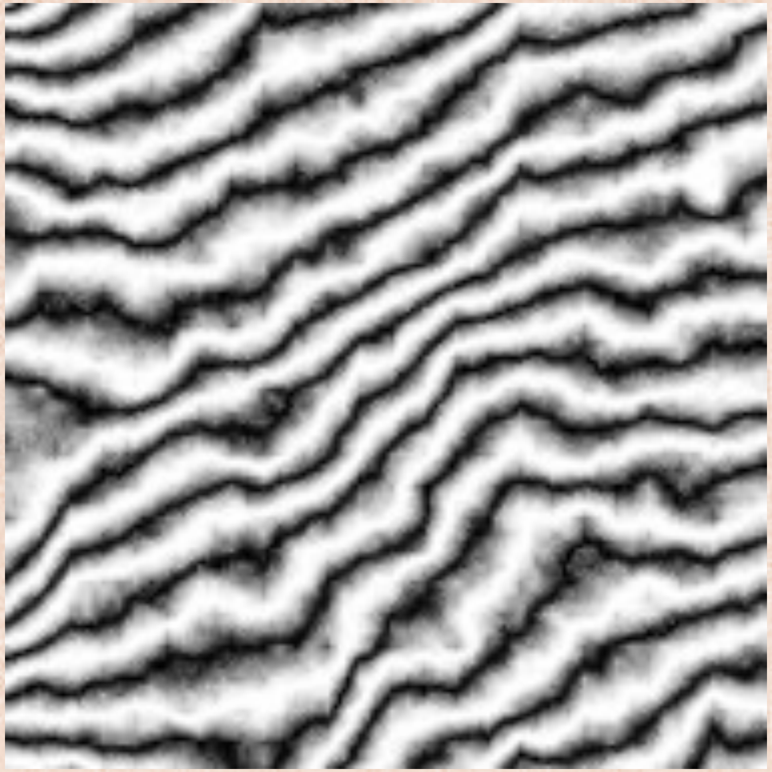


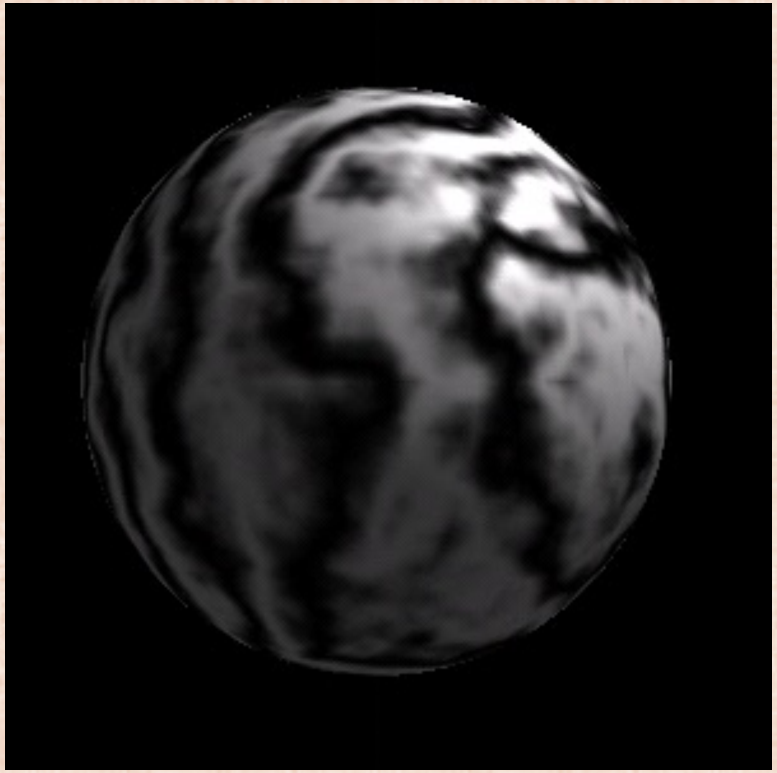
Marble Texture + Perlin Noise

- Set the value of A to scale the amount of noise:

$$marbleColor(u,v) = LayerColor\left(\sin\left(k_u u + k_v v + A * perlin(u,v)\right)\right)$$



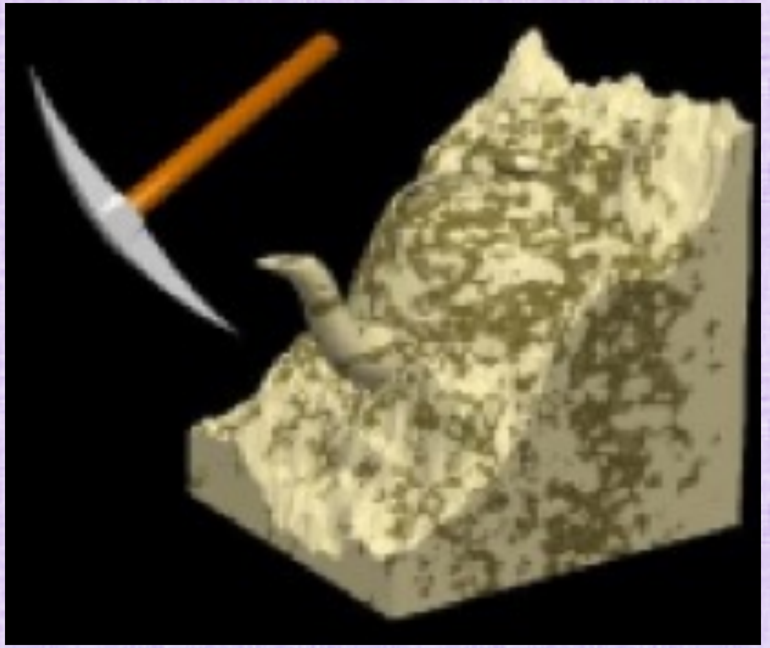


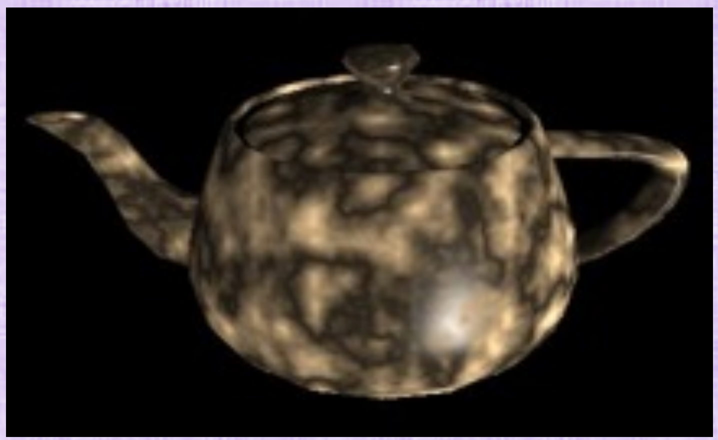


3D Marble Texture

- "Carve" an object out of a 3D texture
- Marble texture function w/Perlin noise (for 3D):

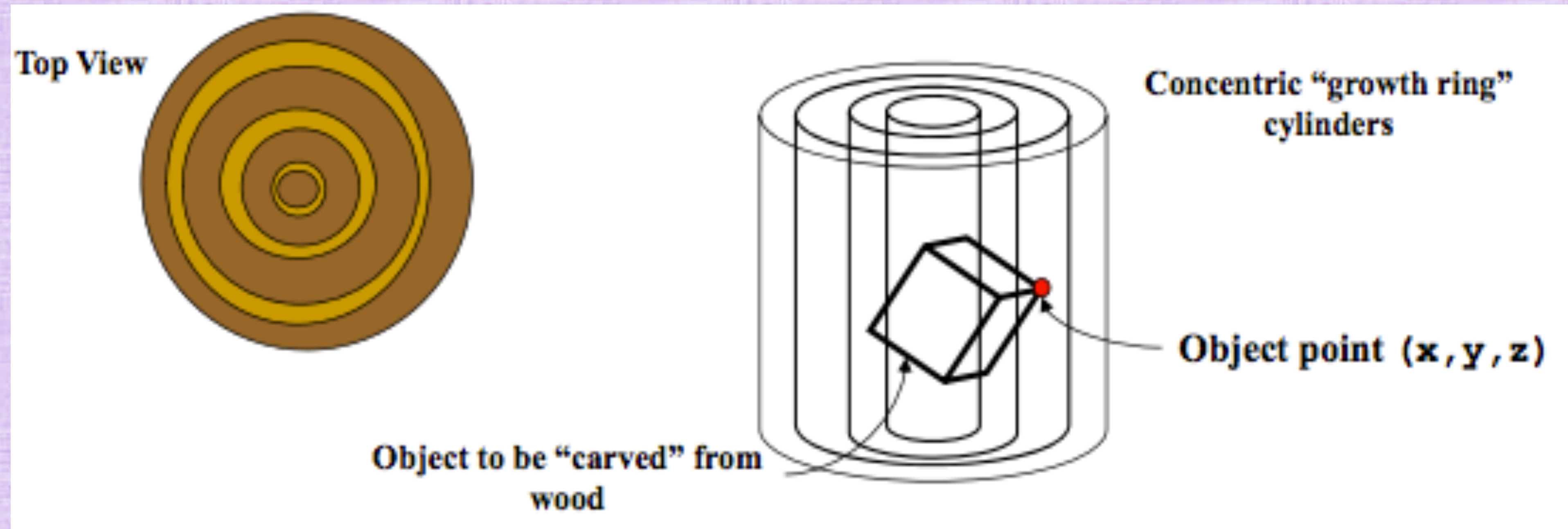
$$marbleColor(u,v,w) = LayerColor\left(\sin\left(k_u u + k_v v + k_w w + A * perlin(u,v,w)\right)\right)$$

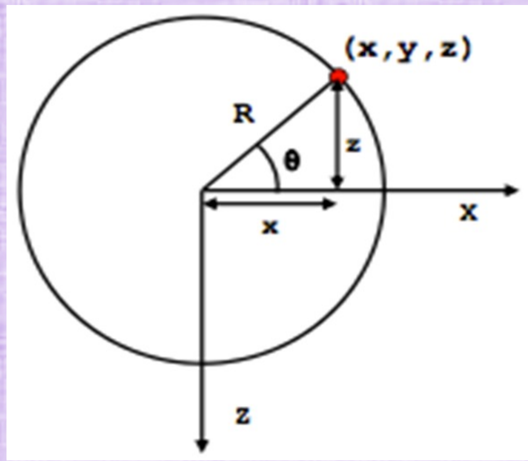




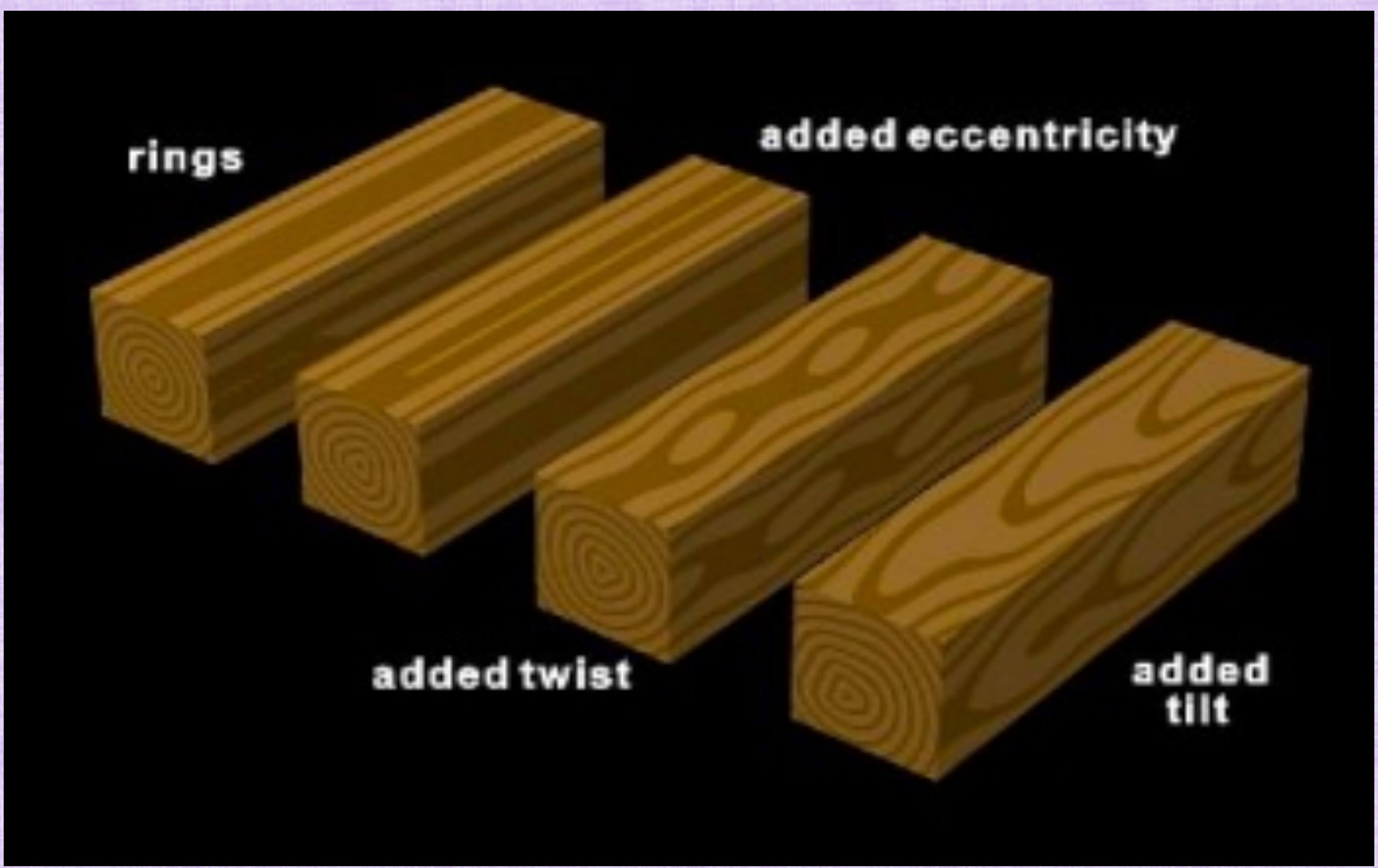
3D Wood Texture

- Procedurally generate tree rings (and cut the object out of the 3D texture)
- Cylindrical coordinates for (x, y, z) object points: $H = y, R = \sqrt{x^2 + z^2}, \theta = \tan^{-1}\left(\frac{z}{x}\right)$





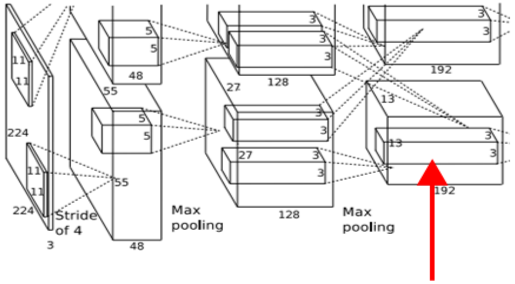
3D Wood Texture

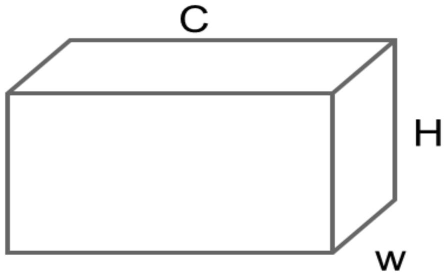


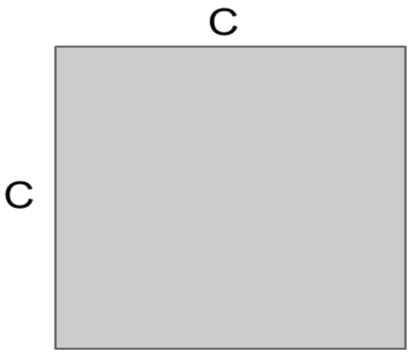
Machine Learning

Neural Texture Synthesis: Gram Matrix

This image is in the public domain.







Each layer of CNN gives C x H x W tensor of features; H x W grid of C-dimensional vectors

Outer product of two C-dimensional vectors gives C x C matrix measuring co-occurrence

Average over all HW pairs of vectors, giving **Gram matrix** of shape C x C

Efficient to compute; reshape features from

C x H x W to =C x HW

then compute $G = FF^T$

Machine Learning

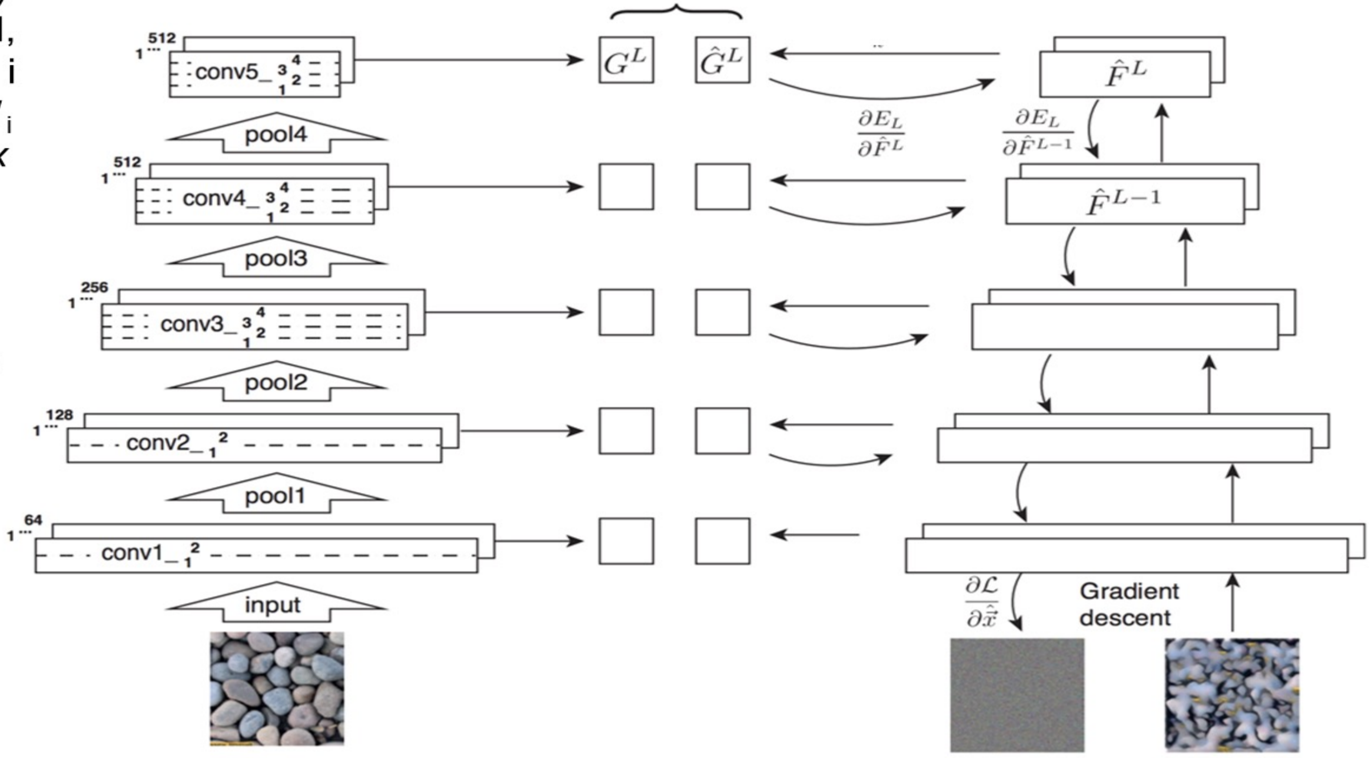
Neural Texture Synthesis

$$E_l = \frac{1}{4N_l^2 M_l^2} \sum_{i,j} \left(G_{ij}^l - \hat{G}_{ij}^l \right)^2 \qquad \mathcal{L}(\vec{x}, \hat{\vec{x}}) = \sum_{l=0}^{L} w_l E_l$$

1. Pretrain a CNN on ImageNet (VGG-19)
2. Run input texture forward through CNN, record activations on every layer; layer i gives feature map of shape $C_i \times H_i \times W_i$
3. At each layer compute the *Gram matrix* giving outer product of features:

$$G_{ij}^l = \sum_k F_{ik}^l F_{jk}^l \quad \text{(shape } C_i \times C_i\text{)}$$

4. Initialize generated image from random noise
5. Pass generated image through CNN, compute Gram matrix on each layer
6. Compute loss: weighted sum of L2 distance between Gram matrices
7. Backprop to get gradient on image
8. Make gradient step on image
9. GOTO 5



Gatys, Ecker, and Bethge, "Texture Synthesis Using Convolutional Neural Networks", NIPS 2015

Machine Learning

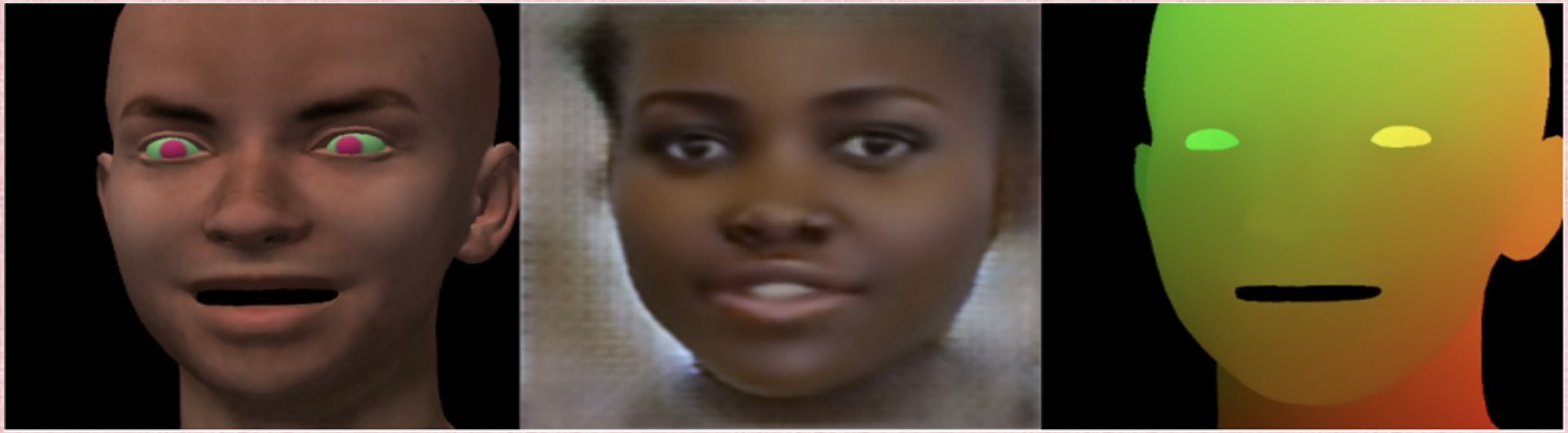
Neural Texture Synthesis

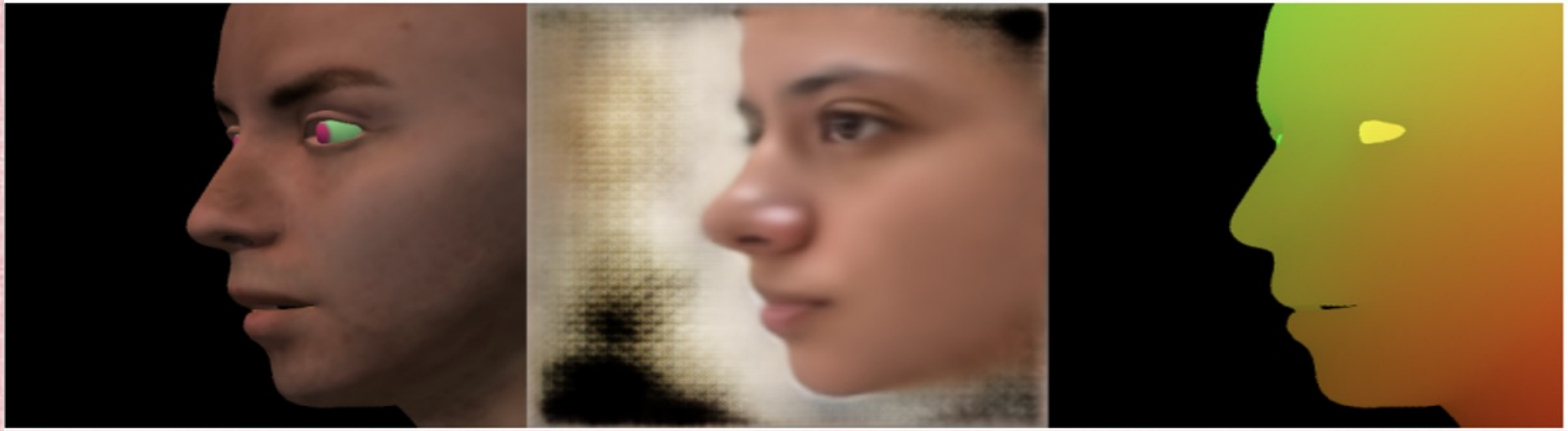
Reconstructing texture from higher layers recovers larger features from the input texture

Gatys, Ecker, and Bethge, "Texture Synthesis Using Convolutional Neural Networks", NIPS 2015

Machine Learning

Machine Learning





Machine Learning

elon musk in a space suit

3d animation